2.1.Lorentz-Transformations

From Einstein, Mein Weltbild

1.) Newton: He Introduces: Absolute space and absolute time

Newton realizes: Only distances in space and time are not enough!

- Newton: Realizes that space has a kind of reality!

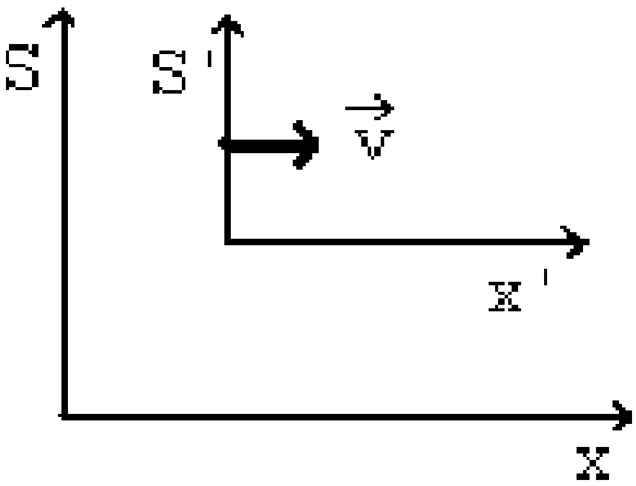
2.) Newton: Introduction instantaneous forces in his gravitation theory

Newton: Gravitation interaction not the final answer

3.) Newton: Weight and inertia have the
same mass, he finds it
strange

Postulate of Einstein:

The laws of nature have the same shape
in all inertial systems



Galilei-Transformation: x'=x-v· t

Possibility of a different scale of time:
x=x'+v ·t'

The scale of length may change!

$$x' = \gamma \cdot (x - v \cdot t)$$ a

γ : independent of space and time, but $\gamma(v)$

$$x = \gamma \cdot (x' + v \cdot t')$$ b

Constraint:

For v $\longrightarrow 0, \gamma \rightarrow 1$

At x=0, t=0 be x = 0 and t =0
Flash into x- direction, light propagation with c in all reference systems!

Put now: $x = c \cdot t$ and $x' = c\,t'$

a (1) $c \cdot t' = \gamma \cdot (c - v) \cdot t$

b (2) $c \cdot t = \gamma \cdot (c + v) \cdot t'$

$$\frac{(1)\cdot(2)}{t \cdot t'} :\Rightarrow \; c^2 = \gamma^2 \cdot (c^2 - v^2) \Rightarrow \gamma = \pm \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow$$

$$x' = \frac{x - v \cdot t}{\sqrt{1-\frac{v^2}{c^2}}};\; x = \frac{x' + v \cdot t'}{\sqrt{1-\frac{v^2}{c^2}}}$$

Lorentz-Transformations

$$t' = \frac{t - \frac{v \cdot x}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}};\; t = \frac{t' + \frac{v \cdot x'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

For $v \ll c \Rightarrow t = t'$

and $x = x' + v \cdot t$

$t'(x)$: No universal time

Postulate of Einstein can be expressed to:

All laws of physics must be invariant by carrying out Lorentz-Transformations

Addition of velocity:

$$x' = \frac{x - v\cdot t}{\sqrt{1-\frac{v^2}{c^2}}}; x = \frac{x' + v\cdot t'}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \Delta x = \frac{\Delta x' + v\cdot\Delta t'}{\sqrt{1-\frac{v^2}{c^2}}}; \Delta t = \frac{\Delta t' + \frac{v\cdot\Delta x'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t' = \frac{t - \frac{v\cdot x}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}; t = \frac{t' + \frac{v\cdot x'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \frac{\Delta x}{\Delta t} = \frac{\Delta x' + v\cdot\Delta t'}{\Delta t' + \frac{v\cdot\Delta x'}{c^2}} = \frac{\left(\frac{\Delta x'}{\Delta t'}\right)+v}{1+\left(\frac{v}{c^2}\right)\left(\frac{\Delta x'}{\Delta t'}\right)}$$

or with $\frac{\Delta x}{\Delta t} = u$ and $\frac{\Delta x'}{\Delta t'} = u' \Rightarrow u = \frac{u'+v}{1+\frac{u'\cdot v}{c^2}}$

Difference to Galilei-Transformation: $\frac{1}{1+\frac{u'\cdot v}{c^2}}$

Length depends on movement:

$$x_2 - x_1 = \frac{x_2' - x_1'}{\sqrt{1-\frac{v^2}{c^2}}};$$

Contraction of lengths!

i.e.: Elektron with 3.5 GeV energy flies 165 m through lab.system:

Längenmessung $x_2' - x_1'$

$t_1' = t_2'$

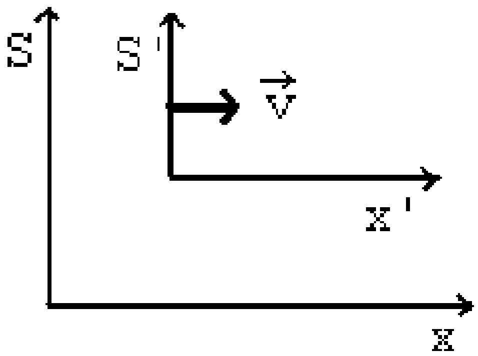
In **its** restsystem this are 2.36cm

$$\frac{v}{c} = 0.99985$$

Contraction only in direction of the relative movement:

Dilatation of time: $\Delta t' = \dfrac{\Delta t - (\frac{v}{c^2}) \cdot \Delta x}{\sqrt{1-\frac{v^2}{c^2}}}$; $\Delta x = 0$,

because the clock rest in S



$\Delta t' = \dfrac{\Delta t}{\sqrt{1-\frac{v^2}{c^2}}}$: Dilatation of time!

Example: Decay of a myon in Atmosphere

Lifetime in rest system $\approx 2 \cdot 10^{-6} s$

Lifetime in the moving system: $141 \cdot 10^{-6} s$

It covers with almost speed of light

$$141 \cdot 10^{-6} \cdot 3 \cdot 10^{5} = 42.\,3 km$$

Relativistic Mechanics:

Conditions: 1. Relativistic momentum and rel. energy are conserved

2. $v \ll c \Rightarrow$ Newtons Mechanics

Def.: Momentum: $\vec{p} = \frac{m \cdot \vec{v}}{\sqrt{1-\frac{v^2}{c^2}}}$; Force: $\frac{d\vec{p}}{dt} = \vec{F}$

Example: $\frac{dp}{dt} = F =$ constant $\Rightarrow \int dp = \int F \cdot dt \Rightarrow$

$$p = F \cdot t = \frac{m \cdot v}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{m^2 \cdot v^2}{F^2 \cdot t^2} = 1 - \frac{v^2}{c^2}; F^2 \cdot t^2 \cdot \frac{v^2}{c^2} + m^2 \cdot v^2 = F^2 \cdot t^2$$

$$\frac{v^2}{c^2} \cdot (F^2 \cdot t^2 + m^2 \cdot c^2) = F^2 \cdot t^2$$

$$\frac{v^2}{c^2} = \frac{\frac{F^2 \cdot t^2}{m^2 \cdot c^2}}{1+\frac{F^2 \cdot t^2}{m^2 \cdot c^2}} \quad \text{or} \quad \frac{v}{c} = \frac{\frac{F \cdot t}{m \cdot c}}{\sqrt{1+\frac{F^2 \cdot t^2}{m^2 \cdot c^2}}}$$

$$\frac{v}{c} = \frac{\frac{F \cdot t}{m \cdot c}}{\sqrt{1+\frac{F^2 \cdot t^2}{m^2 \cdot c^2}}}$$

Limiting cases: $\lim\limits_{\frac{F \cdot t}{m \cdot c} \ll 1} \frac{v}{c} = \frac{F \cdot t}{m \cdot c}$

Non relativistic

$$\frac{F}{m} = a$$

$$\lim_{\frac{F \cdot t}{m \cdot c} \gg 1} \frac{v}{c} = 1 \Rightarrow$$

It does not depend how long force acts the velocity does not exceed c !

$$v = a \cdot t$$

Relativistic Energy:

$$A = \int_0^x F \cdot dx = \int_0^t \frac{dp}{dt} \cdot v \cdot dt = \int_0^p v \cdot dp$$

with s.a. $v = \frac{\frac{p}{m}}{\sqrt{(1+(\frac{p}{m \cdot c})^2)}} \Rightarrow$

$$A = \int_0^p \frac{\frac{p}{m}}{\sqrt{(1+(\frac{p}{m \cdot c})^2)}} dp = \sqrt{\left(\frac{m^2 c^2 + p^2}{m^2 c^2}\right)} mc^2 - mc^2$$

$$p = \gamma \cdot m \cdot v \Rightarrow A = \gamma \cdot mc^2 - mc^2$$

$$v \ll c \Rightarrow A = mc^2 \left[1 + \frac{1}{2} \frac{v^2}{c^2} + \frac{3}{8} \left(\frac{v^2}{c^2}\right)^2 + .. -1 \right] \qquad \Rightarrow A \simeq \frac{1}{2} m \cdot v^2$$

mc^2 : Rest energy

$\gamma \cdot mc^2$: Total energy E

$\gamma \cdot mc^2 - mc^2$: Kinetic energy E_{kin}

At what time E_{kin} =mc ^{2}?

$$\gamma = \frac{2 \cdot m \cdot c^2}{m \cdot c^2} = 2; \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow$$

$$1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2} \Rightarrow \frac{v}{c} = \beta = \sqrt{1 - \frac{1}{\gamma^2}}$$

$$\frac{v}{c} = \sqrt{1 - \frac{1}{2^2}} = \frac{1}{2}\sqrt{3} = .86603$$

$$\boxed{E^2 = (p \cdot c)^2 + (m \cdot c^2)^2}$$

Mechanics: Unit of energy:Joule

Elementary particle : 1 eV $\simeq 1.6 \cdot 10^{-19}$ Joule

Particle with m $\cdot c^2 = 0 \Rightarrow E \cdot \sqrt{1 - \frac{v^2}{c^2}} = m \cdot c^2 = 0 \Rightarrow$ if

$$E \neq 0 \Rightarrow v = c$$

Example for mass energy conversion: $E_0 = m \cdot c^2 \Longrightarrow$

Ass.: Complete conversion of mass in energy in one year

1kg:

$$E_0 = 9 \cdot 10^{16} J$$

$$3600 \cdot 24 \cdot 365 = 31\,536\,000 s$$

$$\boxed{\frac{9 \cdot 10^{16}}{31\,536\,000} = 2.8539 \times 10^9 = 2.8539 \times 10^3 MW}$$

Conversions in reactions:

	$\frac{\Delta m}{m}$
Chemisty	$\sim 10^{-9}$
Nuclei	$\backsim 10^{-3}$
Fission	$\backsim 10^{-3}$
Fusion	$\backsim 6 \cdot 10^{-3}$
Annihilation	$=1$

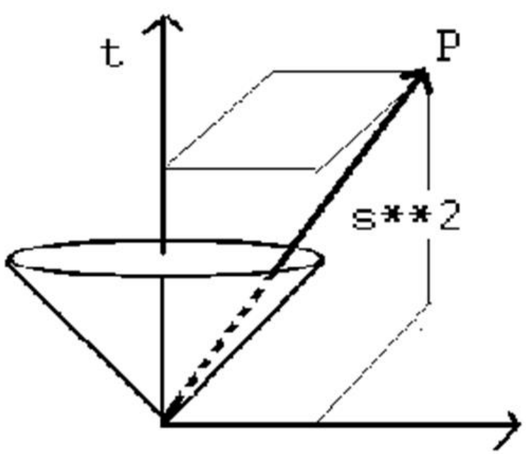
Outcome of the Lorentz transformations: World lines have to lie within the light cone

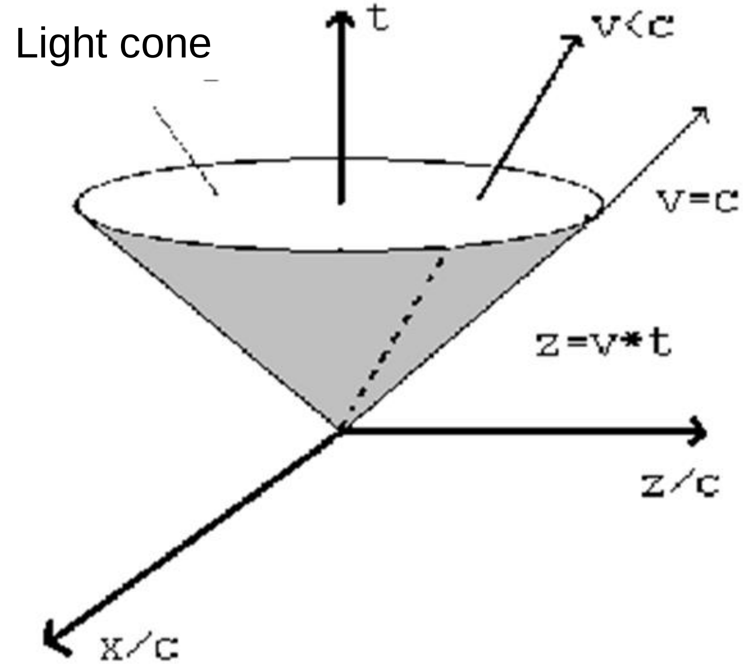
Geometry of Euklid:

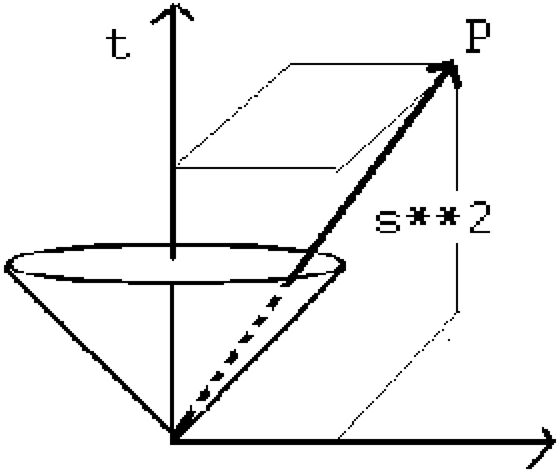
$$r^2 = x^2 + y^2 + z^2$$

Minkowski geometry:

$$s^2 = t^2 - \left(\frac{x}{c}\right)^2 - \left(\frac{y}{c}\right)^2 - \left(\frac{z}{c}\right)^2$$

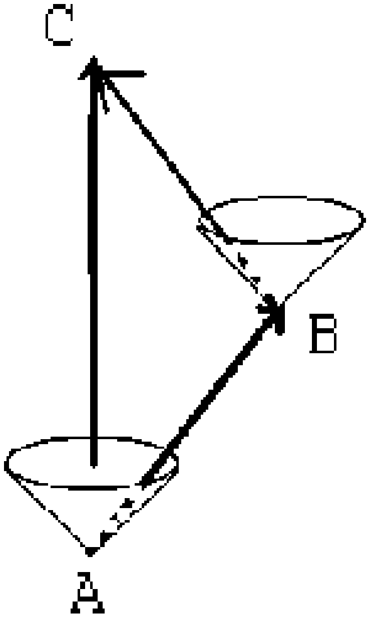






$$s^2 = t^2 - \left(\frac{x}{c}\right)^2 - \left(\frac{y}{c}\right)^2 - \left(\frac{z}{c}\right)^2$$

Twin paradox:



AC: Twin on earth
ABC: Twin underway

Minkowski: AC > AB+BC

Geometry of Euklied: AC < AB+BC