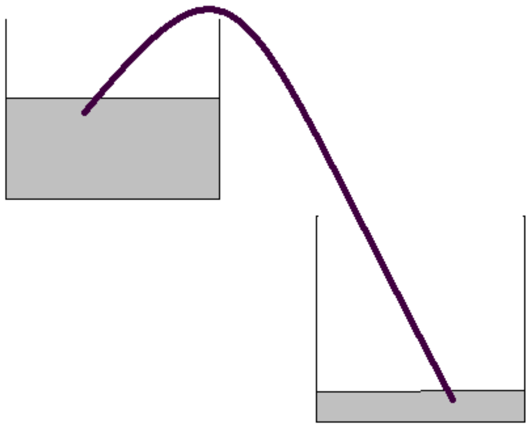
3.4. Surface tension and capillarity

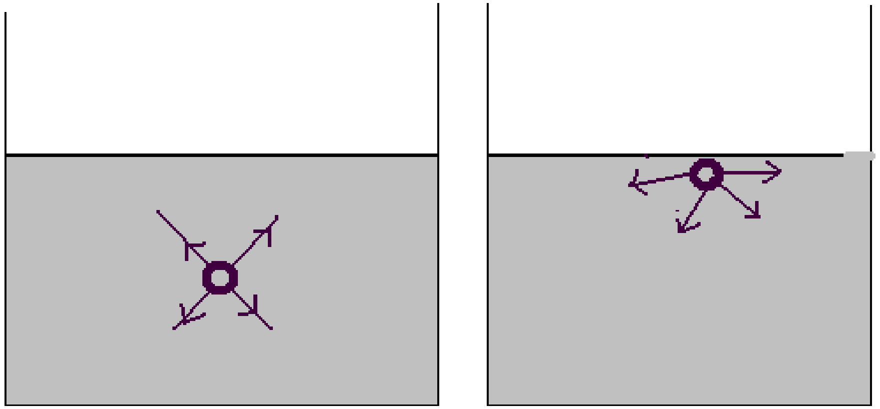
From experiment: liquid thread, molecules of

the liquid show cohesion.



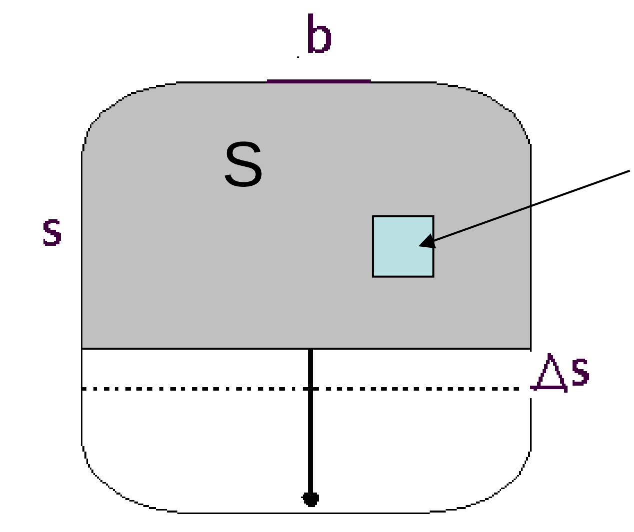
Properties of condensed matter:
Between molecules dominate
strong forces of short distances, 10^{-7}cm
practically up to the next neighbors.

Inside of the liquid: The forces cancel each other



On the surface:
It exists a resultant force towards inside.

At an increase surface: Against the forces , which pull towards inside, work ΔW has to be provided.



Refering to unit of area ΔS, a surface tension will be defined:

a surface tension

$$\sigma_S = \frac{\Delta W}{\Delta S} \quad \text{with} \quad \sigma_S \; (N/m)$$

related to a unit surface unity

Example: Lamella of soap

Surface: S=2*s*b

By a shift of the holder:

$$\Delta S = 2 \cdot b \cdot \Delta s$$ with increase of energy: $$= \sigma_S = 2b \cdot \Delta_s$$

$$\Delta W = \sigma_S \cdot \Delta s = \sigma_s \cdot 2b \cdot \Delta s$$ Work done

$$\Delta W = F \cdot \Delta s$$ and thus $$\sigma_s = \frac{F}{2b}$$

$\sigma_S = \frac{\Delta W}{\Delta S}$ __ with _$[\sigma_S] = \frac{N}{m}$

Example: Lamella of soap

Surface: $S = 2 \cdot s \cdot b$

Work done:

$\Delta W = F \cdot \Delta s$ and thus $\sigma_S = \frac{F}{2b}$

Surfaces of liqids are , minimal surfaces i.e. at a given volume, lowest surface

With increase of energy

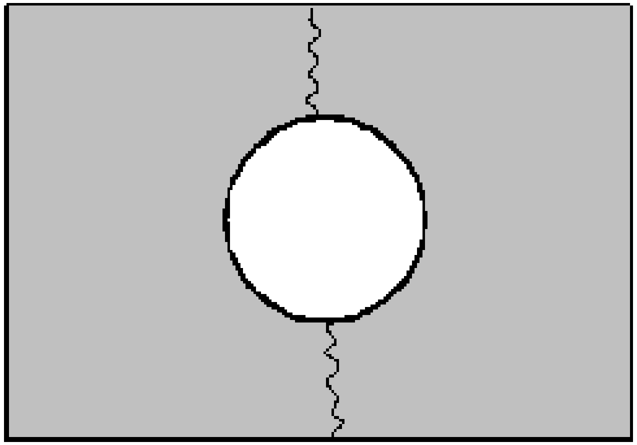
$$\Delta W = \sigma_S \cdot 2b \cdot \Delta s$$

For water one has:

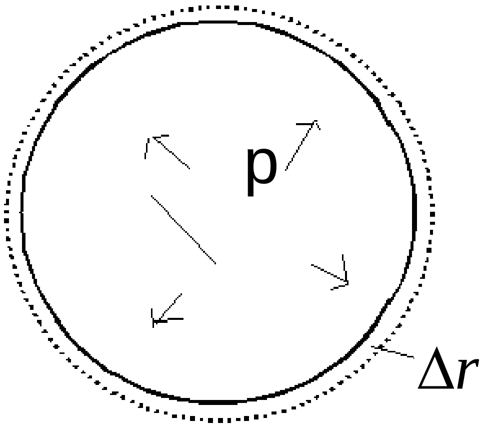
$$\sigma_S = 0.073 N/m$$

For mercury:

$$\sigma_S = 0.47 N/m$$



Compression ratios in a soap bubble



Surface energy:

$$W_S = 2 \cdot \sigma_S \cdot 4 \cdot \pi \cdot r^2 = 8 \cdot \pi \cdot \sigma_S \cdot r^2$$

Smaller surfaces mean

A win of energy!

$$\Delta W_S = 16 \cdot \pi \cdot \sigma_S \cdot r \cdot \boxed{\Delta r}$$

> Increase of pressure inside

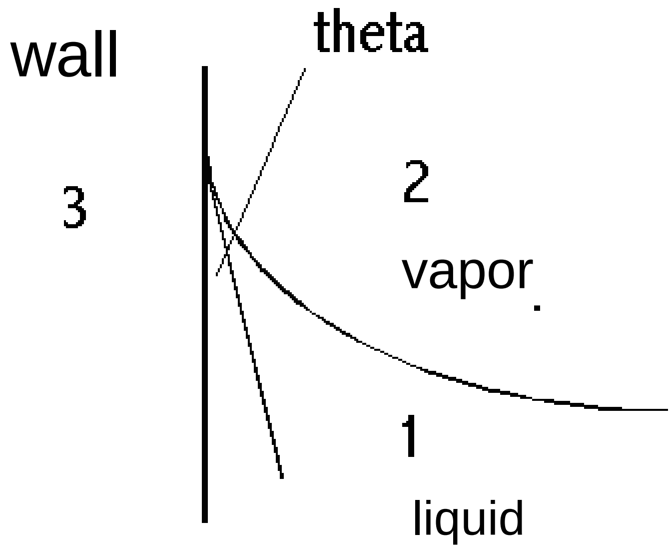
Work done: $\Delta W_P = F \cdot \Delta r = \boxed{p} \cdot S \cdot \Delta r$

Beeing in balance: $\Delta W_S = \Delta W_P$

$$\boxed{p} = 4 \cdot \sigma_S / r$$

bubble smaller, pressure inside larger

Boundary surfaces: Up until now: Liquid - Gas
There are also boundary surfaces liquid-liquid, liquid - all solid



Win of enrgy by coating of boundary surfaces

Balance:

$$\underbrace{\sigma_{23} - \sigma_{12}} = \sigma_{12} \cdot \cos\theta$$

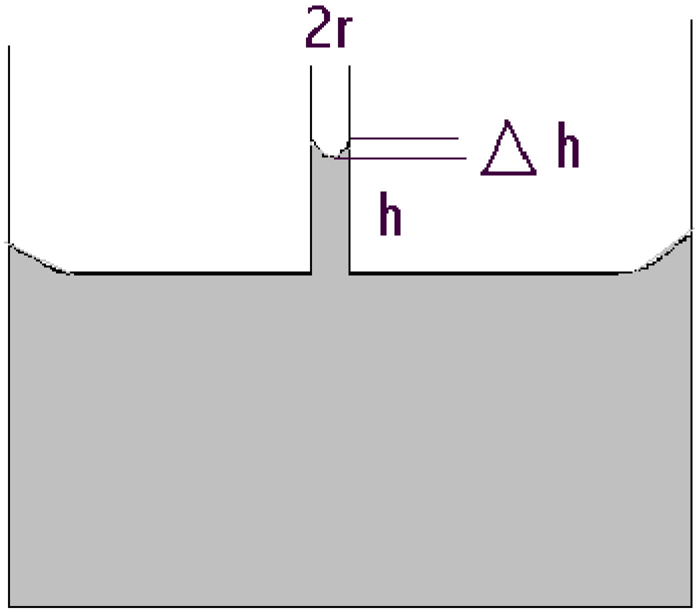
adhesive tension: σ_H

No balance : $\sigma_H \succ \sigma_{12}$:
the liquid crawls up the wall

$0 \leq \sigma_H \leq \sigma_{12}$: coating "*hydrophil*"

$\sigma_H \prec 0$: no coating "*hydrophob*"

Capillarity:



Energy gain by wetting of a surface of a cylinder

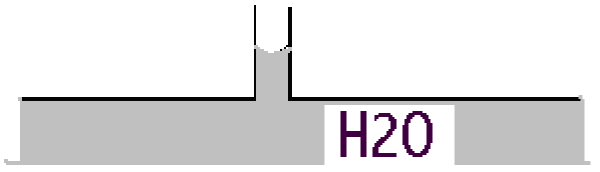
$$\Delta W_H = \sigma_H \cdot 2\pi \cdot r \cdot \Delta h$$

The work at lifting:

$$\Delta W_g = m \cdot g \cdot \Delta h$$

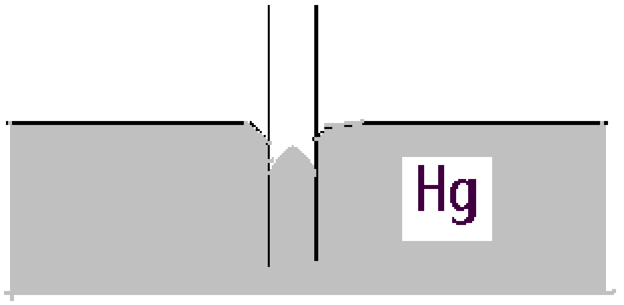
$$= \varrho \cdot V \cdot g \cdot \Delta h = \varrho \cdot g \cdot \Delta h \cdot \pi \cdot r^2 \cdot h$$

balance : $\Delta W_H = \Delta W_g$



Capillarity height:

$$h = \boxed{2} \cdot \sigma_H / \varrho \cdot g \cdot r$$



3.5. Movements of liquids and gases

Uniform treatment of gases and liquids:

Velocity $\ll$ Velocity of sound: common term:

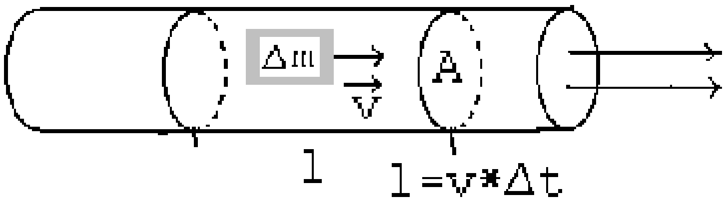
Fluid. Three kinds of currents

a) *Ideal liquid: No forces of friction*

b) *Viscous flow : Friction dominates*

c) *Turbulent flow of real liquids*

a) **Transport of liquids:**



Flux:

$$\Phi = \frac{\text{mass of liquid entering through A in } \Delta t}{\text{time interval}}$$

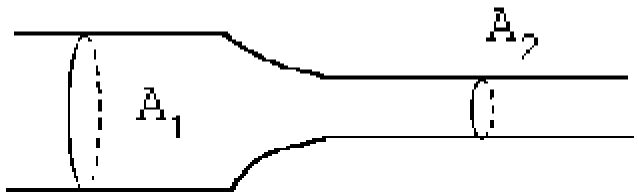
$$\Phi = \frac{m(A,\Delta t,v)}{\Delta t} = \frac{\rho \cdot V(A,\Delta t,v)}{\Delta t} = \frac{\rho \cdot A \cdot v \cdot \Delta t}{\Delta t} = \rho \cdot A \cdot v$$

v : Not necessarely constant

$$\vec{j} = \rho \cdot \vec{v}$$

Density of current: Flux/Area

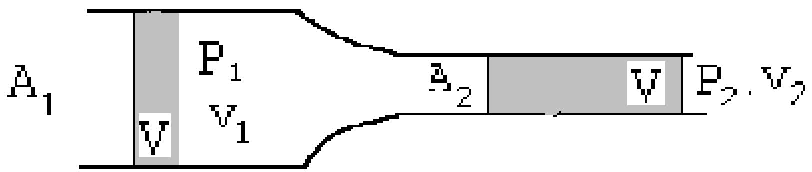
Principle of continuity: Flux remains preserved:



$$\Phi = \text{constant}$$

$$\Phi_1 = \Phi_2$$

$$\Rightarrow A_1 \cdot v_1 = A_2 \cdot v_2 \Rightarrow \frac{v_1}{v_2} = \frac{A_2}{A_1}$$



Volume of liquid wins kinetic energy :

$$\Delta W_{kin} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}\rho \cdot V \cdot (v_2^2 - v_1^2)$$

In return work needs to be done!

V gets pressed into a pipe

$$T \approx \frac{R}{\nu}$$

$$P_2 \cdot V$$

$$v_1 \prec v_2 \Rightarrow \Delta W = P_1 \cdot V - P_2 \cdot V = (P_1 - P_2) \cdot V$$

$$\Delta W = P_1 \cdot V - P_2 \cdot V = (P_1 - P_2) \cdot V$$

Law of conservation of energy:

$$\Delta W = \Delta W_{kin}$$

$$P_1 \cdot V + \frac{1}{2}\rho \cdot V \cdot v_1^2 = P_2 \cdot V + \frac{1}{2}\rho \cdot V \cdot v_2^2$$

$$\Rightarrow P_1 + \frac{1}{2}\rho \cdot v_1^2 = P_2 + \frac{1}{2}\rho \cdot v_2^2$$

Is valid along of a arbitrarely formed pipe:

$$p + \frac{1}{2}\rho \cdot v^2 = \text{constant} = P_0$$

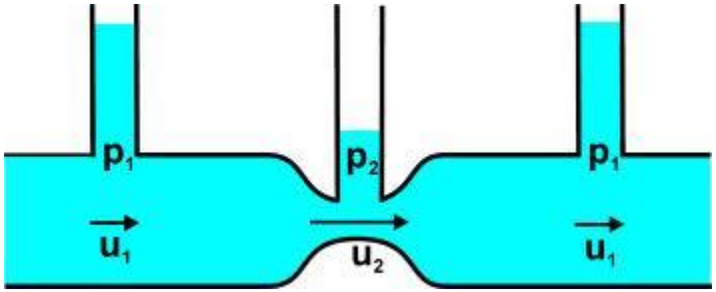
Equation of Bernoulli :

Static pressure+dynamic pressure= total pressure

Huge practical importance $v \sim \frac{1}{A} \Rightarrow$

Static pressure large, if A large, dynamic pressure large, if A small

Hydrodynamic Paradoxon

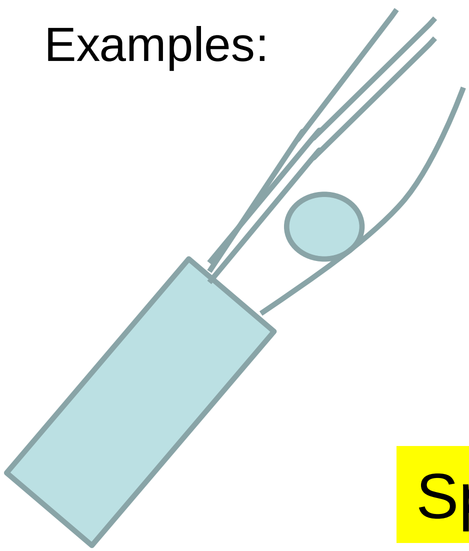


Accordingly:

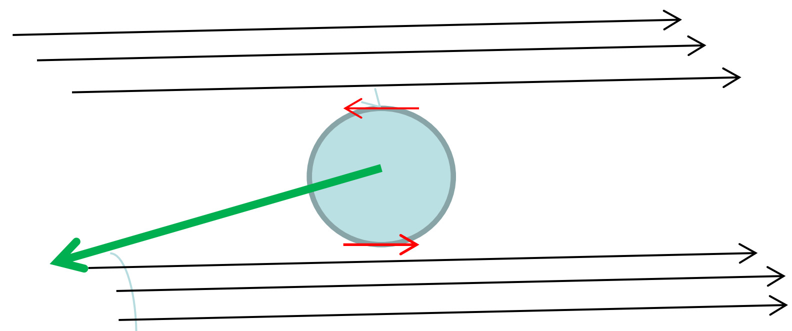
$$p + \frac{1}{2}\rho \cdot v^2 = P_0$$

„small“, if v large

Examples:



Magnuseffect



Sport: Ball „bending“

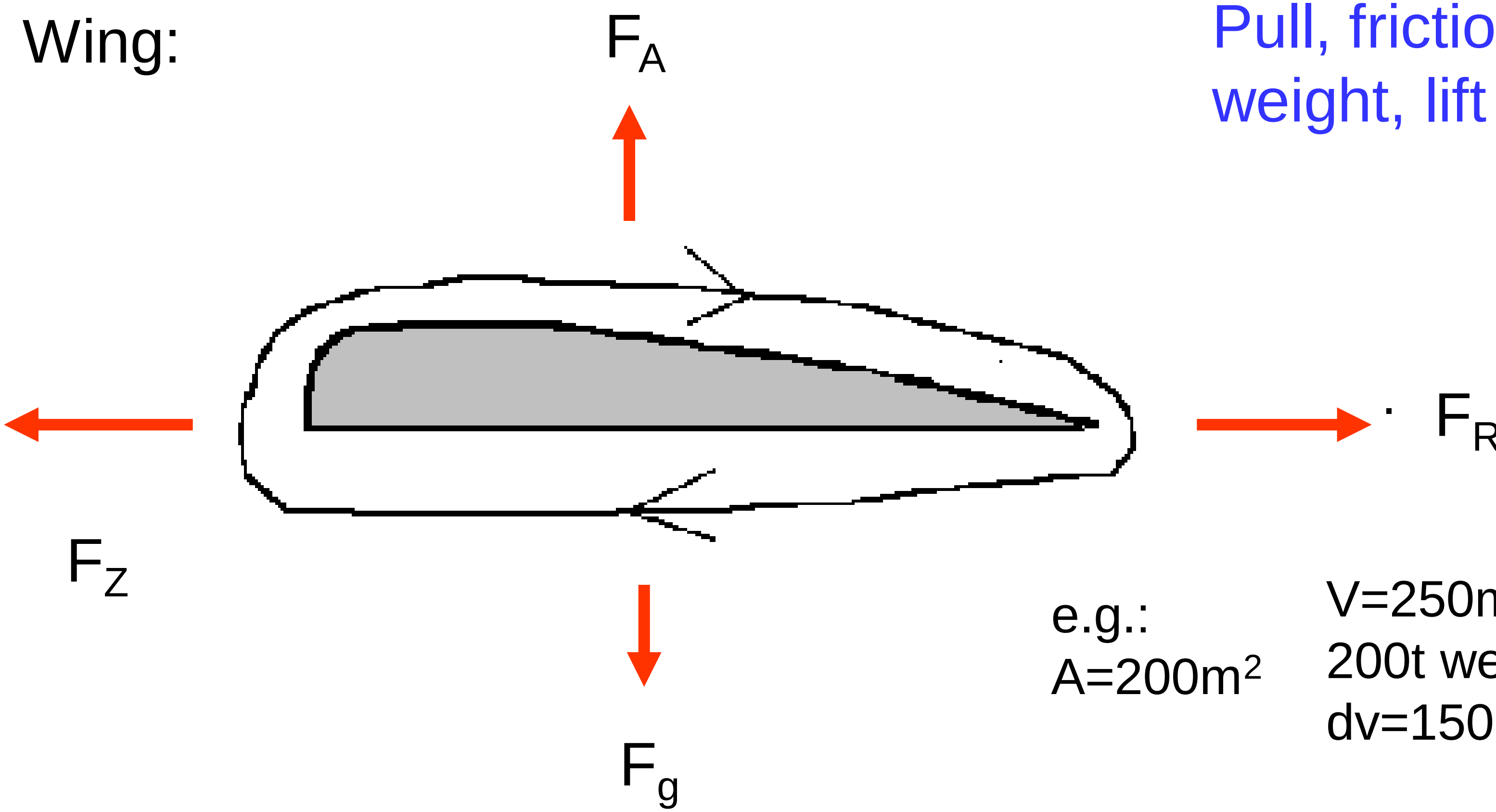
Tennis, table tennis, soccer etc.

Consequence:
Deflection

Rotating surface of the balls reduces/increases
via friction the velocity of air around the surface

Another example:

Wing:



Forces:
Pull, friction
weight, lift

e.g.:
A=200m²

V=250m/s
200t weight.
dv=150km/h

$$F_A = A \cdot 0.5 \cdot \rho \cdot (v_{above}^2 - v_{below}^2)$$

$$F_A = A \cdot 0.5 \cdot \rho \cdot (v_{above} - v_{below}) \cdot (v_{above} + v_{below})$$

or with

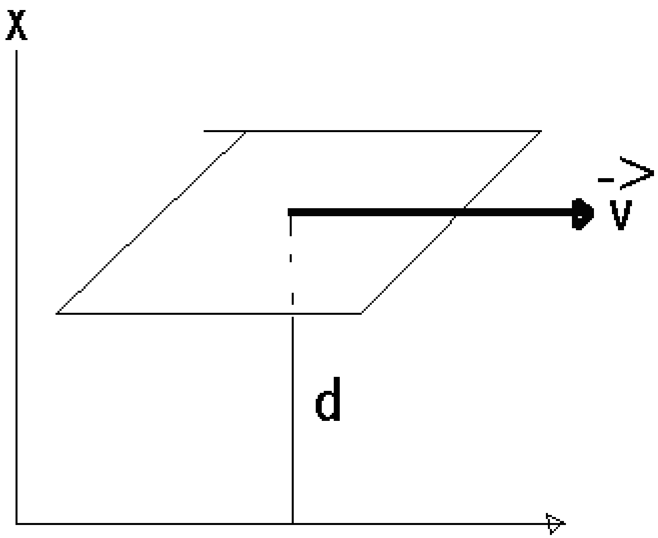
$$(v_{above} + v_{below}) \simeq 2 \cdot v$$

Carrying force:

$$F_A \simeq A \cdot \rho \cdot v \cdot (v_{oben} - v_{unten})$$

b)Laminary flow of viscous liquid

Liquid can not absorb shear stress.



Movable plate with surface S
In order to move a plate above a layer of liquid one needs a force:

$$F \sim S \bullet v/d$$

Interpretation: On a liquid act a kind „shear tension “

Viscosity

$$\tau\ :\ \tau = F/S\_\_d.h\ :\ \_\tau = \eta \bullet v/d \quad [\tau] = N \bullet m/S$$

A sticking layer of liquid slides over the below lying layer etc.

v/d Stands fo a velocity gradient $\rightarrow \frac{dv}{dx}$ *e.g.: Gradient*

$$\eta(T)\ :\ T = temperature, e.g\ :\ H_2O$$

$$\eta(0) = 0.0018, \eta(100) = 0.00029$$

Example: Laminary flux between parallel plates, with v= constant

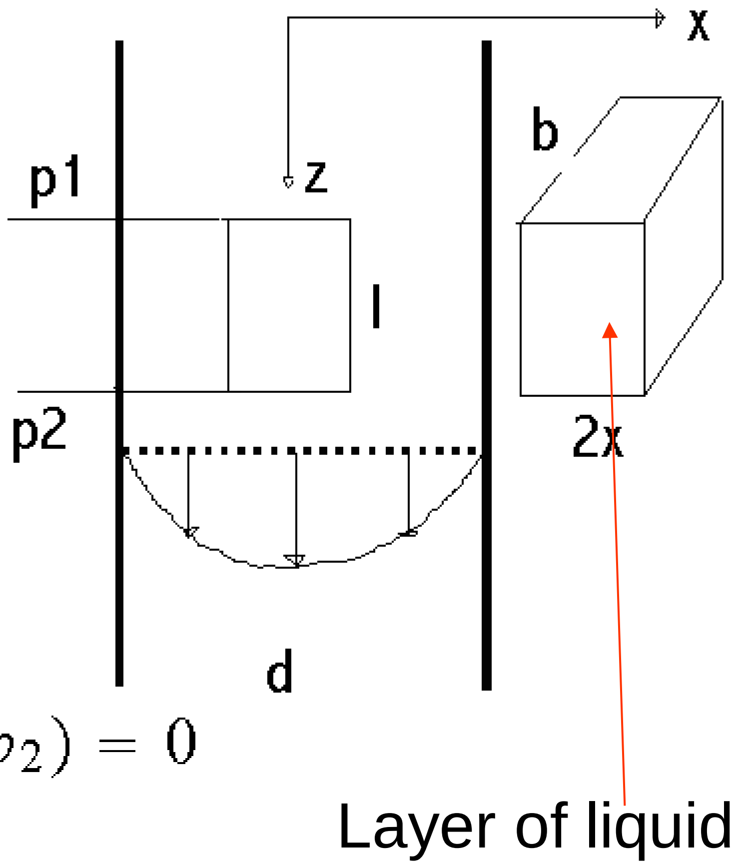
At surfaces: Forces

$$2 \cdot x \cdot b \cdot p_1 - 2 \cdot x \cdot b \cdot p_2$$

At wings: friction forces:

$$\eta \cdot 2 \cdot b \cdot l \cdot dv/dx$$

Constant velocity means:
The sum of all forces =0!



$$\eta \cdot 2 \cdot b \cdot l \cdot dv/dx + 2 \cdot x \cdot b \cdot (p_1 - p_2) = 0$$

$$\rightarrow dv/dx = -x(p_1 - p_2)/(\eta \cdot l)$$

Integration: $v = -x^2 \cdot (p_1 - p_2)/(2 \cdot \eta \cdot l) + c$

$$v = -x^2 \cdot (p_1 - p_2)/(2 \cdot \eta \cdot l) + c$$

with v=0 for x= d/2
yields c to:

C integration constant

$$c = (d/2)^2 \cdot (p_1 - p_2)/(2 \cdot \eta \cdot l)$$

and following:

$$v = (p_1 - p_2)/(2 \cdot \eta \cdot l)((d/2)^2 - x^2)$$

$$(p_1 - p_2)/l = dp/dz$$

Stands for a gradient of pressure

$$\vec{v} \approx grad(p) \cdot ((d/2)^2 - x^2)$$

The tips of the vectors of velocity
lie on a parabola!

Laminary flux via a pipe can be dealt with
in analogy of the flat problem

$$r\,\eta \cdot 2 \cdot \pi \cdot l \cdot dv \cdot /dr + r^2 \cdot \pi \cdot (p_1 - p_2) = 0$$

$$dv/dr = -r \cdot (p_1 - p_2)/(\eta \cdot 2 \cdot l)$$

with integration:

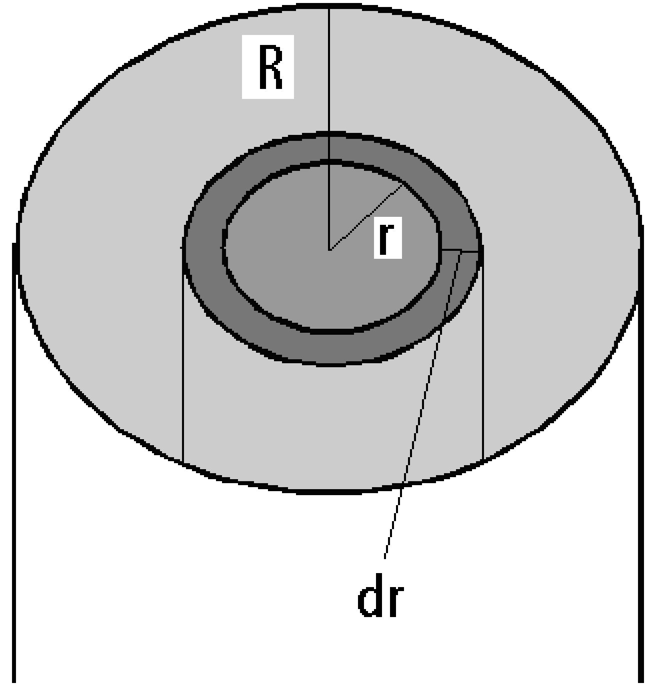
$$-4 \cdot \eta \cdot l \cdot v/(p_1 - p_2) = r^2 + c$$

$$c = -R^2\_für\_v = 0\_an\_r = R$$

$$R^2 - r^2 = 4 \cdot \eta \cdot l \cdot v/(p_1 - p_2)$$

or $$v = (p_1 - p_2) \cdot (R^2 - r^2)/4 \cdot \eta \cdot l$$

Paraboloid



Drifty mass of liquid for a time t:

$$dV = 2 \cdot \pi \cdot r \cdot dr \cdot v \cdot t =$$

$$2\,r\,\pi \cdot dr \cdot (p_1 - p_2) \cdot (R^2 - r^2) \cdot t/4 \cdot \eta \cdot l$$

$$V = \frac{\pi \cdot (p_1 - p_2) \cdot t}{2 \cdot \eta \cdot l} \int_0^R (R^2 - r^2) \cdot r \cdot dr$$

$$V = \pi \cdot R^4 \cdot (p_1 - p_2) \cdot t/8 \cdot \eta \cdot l \cdot$$

Flux:

$$\Phi = m/t = \varrho \cdot V/t \rightarrow$$

Equation of Hagen- Poseuille

$$\Phi = \pi \cdot \varrho \cdot R^4 \cdot (p_1 - p_2)/8 \cdot \eta \cdot l$$