C) Turbulent flow of real liquids

Criterion for transition to laminary to turbulent current

Current of **ideal** liquids: **Inertia force ~** $\rho \cdot v^2$

viscous **:forces of friction ~** $\eta \cdot v \cdot d$

d: typical distance

Following forces are considered:

$$d\vec{F}_R = \eta \cdot \Delta\vec{u} \cdot dV \qquad d\vec{F}_P = -grad\ P \cdot dV \qquad d\vec{F}_g = \rho \cdot \vec{g} \cdot dV$$

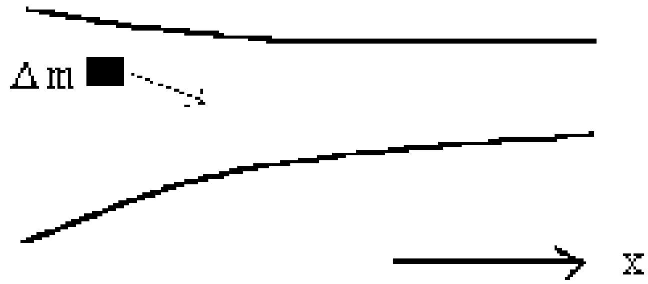
Friction **Force due to pressure** **Gravity**

Those forces spark accelerations: $u \Rightarrow \vec{u} + d\vec{u}$

The total change of velocity u in which element of mass in time dt from r-> r+ dr proceeds

For the x- component:

$$\frac{du_x}{dt} = \frac{\partial u_x}{\partial t} + \frac{\partial u_x}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u_x}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u_x}{\partial z}\frac{\partial z}{\partial t}$$



$\frac{\partial u_x}{\partial t}$: Change on same place

$\frac{\partial u_x}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial u_x}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial u_x}{\partial z}\frac{\partial z}{\partial t}$: Acceleration due to convection

Generally: $\frac{d\vec{u}}{dt} = \frac{\partial \vec{u}}{\partial t} + (\vec{u} \bullet \vec{\nabla})\vec{u} \Longrightarrow$

$$\rho\left(\frac{\partial}{\partial t} + (\vec{u} \bullet \vec{\nabla})\right)\vec{u} = -grad\ P + \rho \bullet \vec{g} + \eta \bullet \Delta\vec{u}$$

Navier-Stokes-equation

Using the vector correletion:

$$(\vec{u} \bullet \vec{\nabla})\vec{u} = \frac{1}{2} grad\ u^2 - (\vec{u} \times rot\, u)$$

On bothe sides rot

With $\eta = 0$

(ideal liquid)

Due to rot(grad P)=0 and rot grad u^2 =0 $\frac{\partial}{\partial t} rot\vec{u} - rot(\vec{u} \times rot\vec{u}) = 0$

Beeing: $rot\ u = 0$ at time t $\Rightarrow \frac{\partial}{\partial t} rot\vec{u} = 0 \Rightarrow$

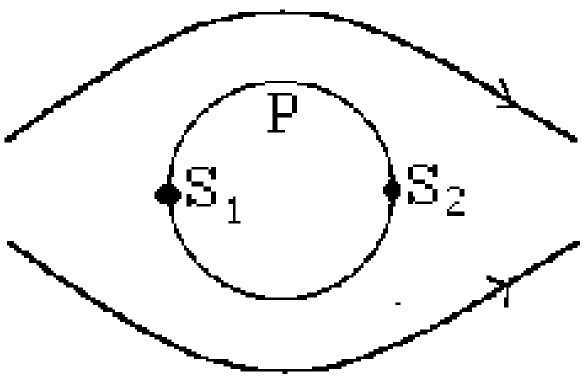
If an ideal liquid without turbulence is set in motion , no to turbulence occurs .

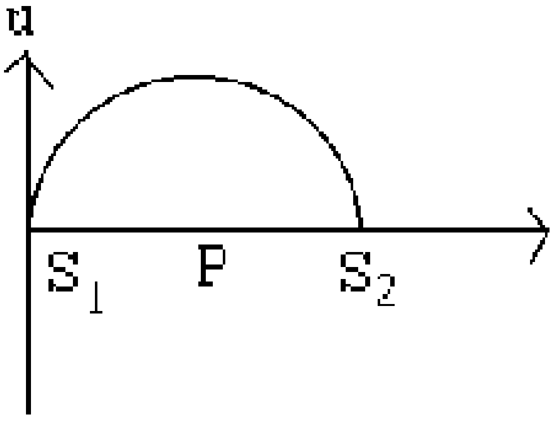
Force of turbulence stays constant $\Rightarrow$ constant in time

Friction is relevant at creation of turbulence . $\Rightarrow$ large „shear force“

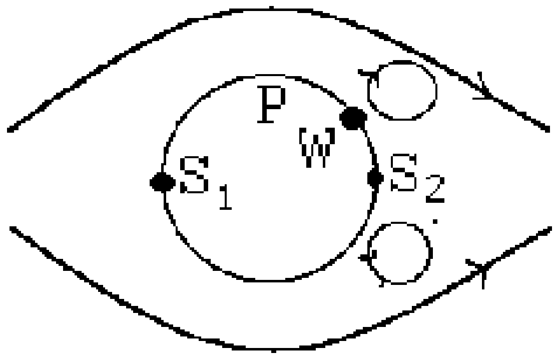
Example: Circulation around a ball

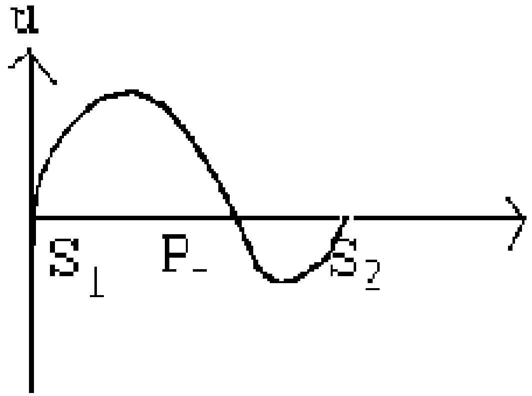
laminary





turbulent

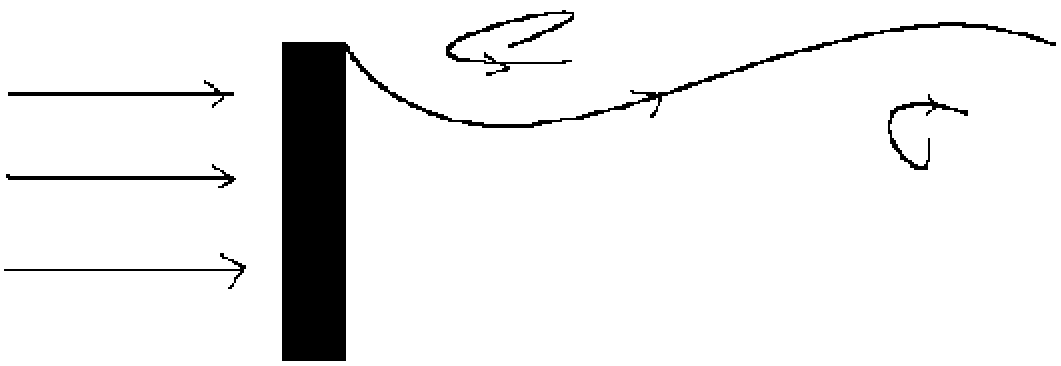




The velocity is reached already at W, which is reached in case of a laminary velocity at S_2 !

$\Longrightarrow$ turbulence

Vortex streets!

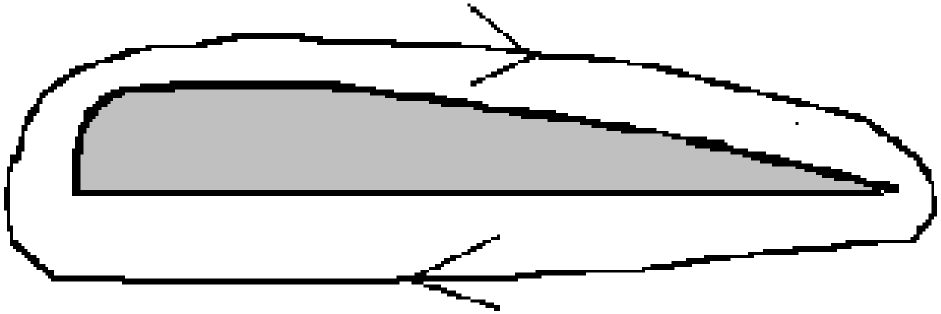


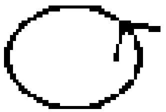
Stay not fixed!

The energy comes from kinetic energy of the flowing medium.

Once again wings!

Vortex $\Rightarrow$ Conservation of angular momentum



laminary flow + 

$\Rightarrow$ lift