Law of similarity and Reynolds number:

Constructional systems of fluid physics can be simulated with smaller models! All dimensions of distance l and time t will be scaled to dimensionen of unity L und T and the velocity expressed as L/T !

$$t = t' \cdot T, \qquad u = u' \cdot \frac{L}{T}, \qquad p = p'\left(\frac{L^2}{T^2}\right) \cdot \rho$$

$$l = l' \cdot L \qquad \nabla = \frac{\nabla'}{L},$$

via $p = \frac{force}{area} = \frac{m \cdot acceleration}{area} = \frac{\rho \cdot L^3 \cdot \frac{L}{T^2}}{L^2}$

The primed quantities are dimensionless!

Navier-Stokes:

$$\rho\left(\frac{\partial}{\partial t} + (\vec{u} \cdot \vec{\nabla})\right)\vec{u} = -grad\, P + \rho \cdot \vec{g} + \eta \cdot \Delta \vec{u}$$

without $\rho \cdot \vec{g}$

$$\left(\frac{\partial}{\partial t} + (\vec{u}' \cdot \vec{\nabla}')\right)\vec{u}' = -\nabla' P' + \frac{1}{\mathrm{Re}} \cdot \Delta' \vec{u}'$$

$$\frac{\rho \cdot L^2}{\eta \cdot T} = \frac{\rho \cdot u \cdot L}{\eta}$$ = Re: Reynolds number

What is the meaning of this number?

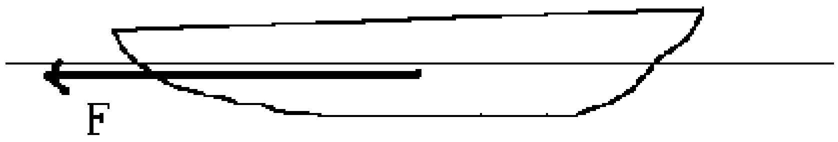
$\frac{\rho \cdot u \cdot L}{\eta}$ Expansion with L^2u: $\frac{\rho \cdot u^2 \cdot L^3}{\eta \cdot L^2 \cdot u} = \boxed{\frac{2 \cdot E_{kin}}{W_{\text{friction}}}} = \text{Re}$

e.g.: from experiment for water pipe: Re=2300

$\gtrsim 2300 \Rightarrow$ turbulent

$\prec 2300 \Rightarrow$ laminary

Other examples, in order to demonstrate the direction of argumentation! Ship in water: Question "Is it more advantageous with respect of fuel economy to build large ships?"



6 Quantities enter a discription: F, ρ, u, η, l, g

Length, Mass, Time

There are 3 relations:

$P = \frac{F}{\rho \cdot u^2 \cdot l^2}$ Coefficient of pressure

Re = $\frac{\rho \cdot u \cdot l}{\eta}$

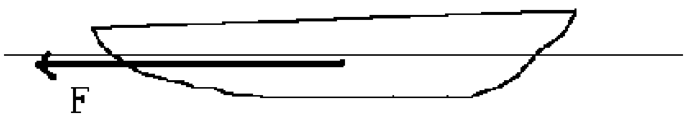
Froudes number (here =0), only for weight waves

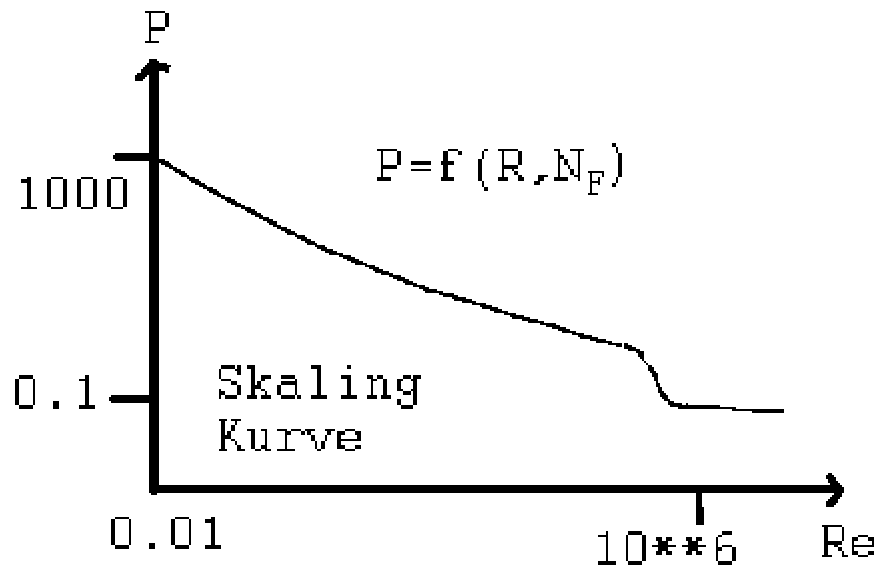
dimensionless: $p = \frac{F}{\varrho \cdot u^2 \cdot l^2}$ proof: $p = \frac{m \cdot \frac{L}{T^2}}{\frac{m}{L^3} \cdot \frac{L^2}{T^2} \cdot L^2}$

$$N_F = \frac{u^2}{l \cdot g}$$

$$F = \rho \cdot u^2 \cdot l^2 \cdot f(\mathrm{Re})$$

weight W~l³





$$\Rightarrow \frac{F}{W} \sim \frac{1}{l} \Rightarrow$$

large ships are more cost efficient

Another example: **Rowing:**

What is the difference of speed of an single compared to an eight rower team?

Power of oarsmans: E=F*u~n, n=number of oarsmen

Weight of a team of oarsmen: W~n R=Re

$$\frac{E}{W} = \frac{\rho \cdot u^3 \cdot l^2 \cdot f(R)}{\rho \cdot l^3} = \frac{u^3 \cdot f(R)}{l}$$

Does not explicitly depend on n!

l, the size of the boat, design for water displacement, depends via $W = \rho \cdot l^3 \sim n$ on n.

$$\Rightarrow l^3 \sim n \Rightarrow l \sim \sqrt[3]{n}$$

Because E/W does not depend on n follows: $u \sim \sqrt[3]{l} \Rightarrow$

$$u \sim \sqrt[3]{\sqrt[3]{n}} = \sqrt[9]{n}$$

$$\frac{one}{eigth} = \frac{1}{\sqrt[9]{8}} = 0.7937$$

Men

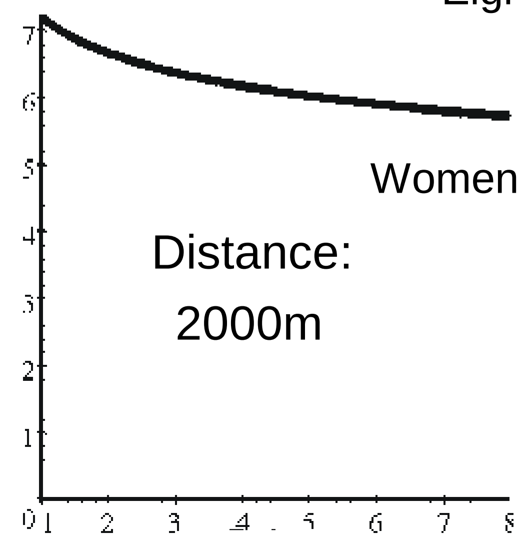
In the race:

0.8117

Women : 0.82

Men: Min.Sec:

Single: 6.33
Eight:5.19



Women: 7.10
5.54

Distance:

2000m

4. Oscillations and waves

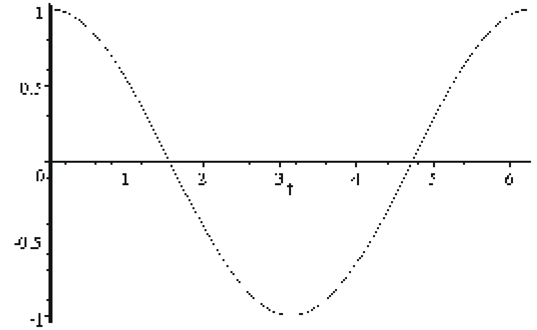
4.1. Free oscillations

a) Flood oscillations

$$m \cdot \ddot{x} = -D \cdot x \Rightarrow \omega_0 = \sqrt{\frac{D}{m}}$$

$m \cdot \ddot{x}$: Inertial force $\quad D \cdot x$: Reset force

Result:



b) Rotary oscillations

$$I \cdot \ddot{\varphi} = -D^* \cdot \varphi \Rightarrow \omega_0 = \sqrt{\frac{D^*}{I}}$$

Damped oscillations:

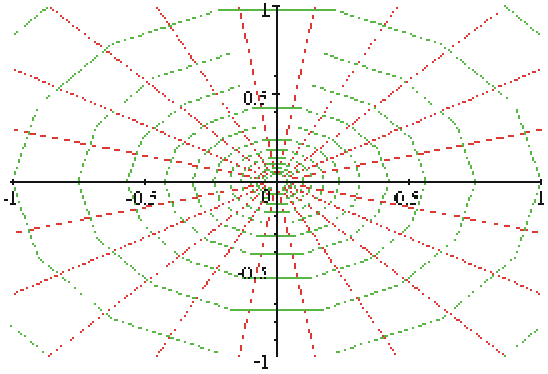
$$m \cdot \ddot{x} = -k \cdot \dot{x} - D \cdot x \quad \text{or} \quad m \cdot \ddot{x} + k \cdot \dot{x} + D \cdot x = 0$$

$k \cdot \dot{x}$ Frictional force

Mathematical structure : *Homogene, linear differential equation*

Solution with complex numbers: $m \cdot \ddot{z} + k \cdot \dot{z} + D \cdot z = 0$

Ansatz: $z = A \cdot e^{i\gamma t}$ γ : complex



Derivation: $\dot{z} = i \cdot \gamma \cdot A \cdot e^{i\gamma t}$

$$\ddot{z} = (i \cdot \gamma)^2 \cdot A \cdot e^{i\gamma t} = -\gamma^2 \cdot A \cdot e^{i\gamma t} \Rightarrow$$

$$-m \cdot \gamma^2 \cdot A \cdot e^{i\gamma t} + i \cdot \gamma \cdot k \cdot A \cdot e^{i\gamma t} + D \cdot A \cdot e^{i\gamma t} = 0$$

hence the characteristic equation: $-m \cdot \gamma^2 + i \cdot \gamma \cdot k + D = 0$

or $\gamma^2 - i \cdot \gamma \cdot k/m - D/m = 0$

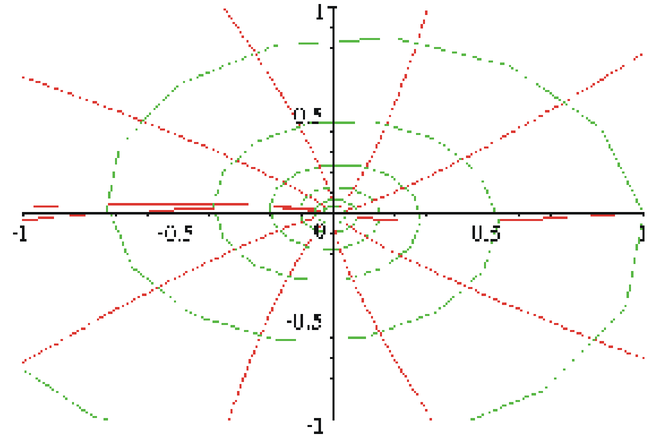
$$\gamma = \frac{1}{2m}\left(ik \pm \sqrt{(-k^2 + 4mD)}\right) = \delta \cdot i \pm \omega \Rightarrow$$

$$z = A \cdot e^{i \cdot (\delta \cdot i \pm \omega)t} = A \cdot e^{-\delta \cdot t} e^{i \cdot \pm \omega t}$$

with δ

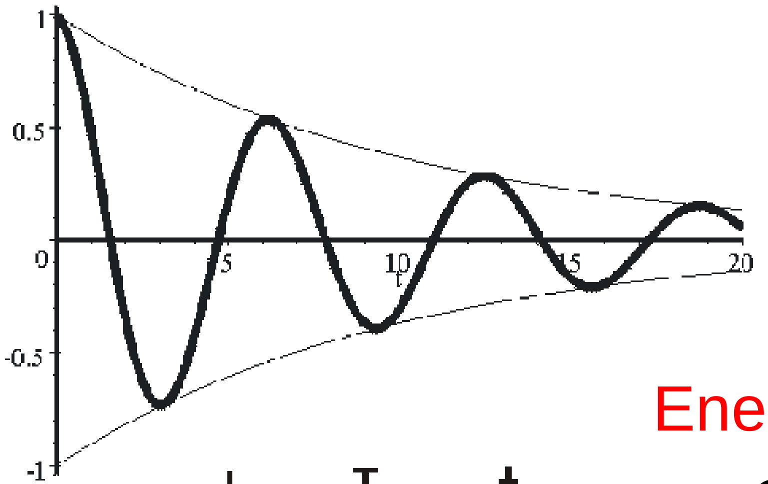
as damping constant!

$$z = A \cdot e^{i\cdot(\delta \cdot i \pm \omega)t} = A \cdot e^{-\delta \cdot t} e^{i \cdot \pm \omega t}$$



Projection on real part:

$$x = x_0 \cdot e^{-\delta \cdot t} \cdot \cos \omega t$$



t_1 T t_2

Without damping

$$W = W_{kin} + W_{pot} = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} D \cdot x^2$$

Energy of damped oscillator:

Oscillates and drops uniformly

From $W = \frac{1}{2} D \cdot x_1^2$ at $x_1(t = 0)$

$W = \frac{1}{2} D \cdot x_2^2$ at $x_2(t = T)$

$$x_2 = x_0 \cdot e^{-\delta \cdot T} \cdot \cos \omega T \Rightarrow W = \frac{1}{2} D \cdot x_0^2 e^{-2\delta \cdot t}$$

i.e.: energy drops twice as much as the amplitude

Definition of figure of merit of a set-up:

$$\frac{2\pi \cdot stored\ energy}{energyloss\ in\ one\ period} = Q = \frac{2\pi \cdot W}{-\dot{W} \cdot T} = \frac{2\pi \cdot W}{2\delta \cdot W \cdot T} = \frac{\pi}{\delta \cdot T} = \frac{\pi \cdot \omega}{\delta \cdot 2\pi} = \frac{\omega}{2\delta}$$

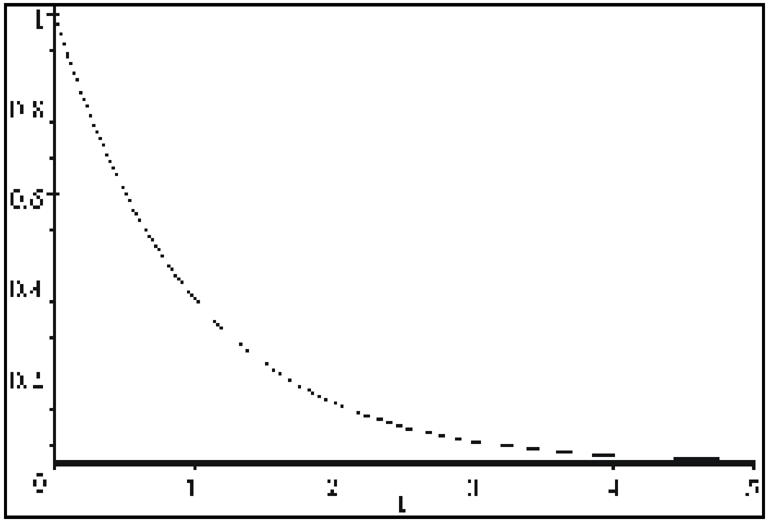
Influence of damping on oscillation:

$$\gamma = \frac{1}{2m}\left(ik \pm \sqrt{(-k^2 + 4mD)}\right)$$

3 cases: a) $k^2 \prec D \cdot m$: Case of oscillation

b) $k^2 = 4 \cdot D \cdot m$ aperiodic limiting case

c) $k^2 \succ 4 \cdot D \cdot m$ crawl speed



ω imaginary :

System returns exponentielly to rest position, as b)

4.2. Forced oscillations

Force from outside F

act on periodically on an oscillatory system.
mathematical treatment : $m \cdot \ddot{z} + k \cdot \dot{z} + D \cdot z = F_0 \cdot e^{i\omega t}$

Ansatz for z: $z = A \cdot e^{i(\omega t - \alpha)}$

α : Phase shift to outside force

Derivatives:

Solution via differential equation

$$\dot{z} = i \cdot \omega \cdot \quad e^{i(\omega t - \alpha)}$$

$$\ddot{z} = -\omega^2 \cdot A \cdot e^{i(\omega t - \alpha)} \Rightarrow$$

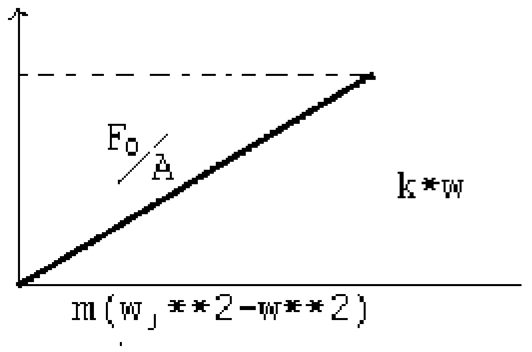
$$-\omega^2 \cdot m \cdot A \cdot e^{i(\omega t - \alpha)} + i \cdot k \cdot \omega \cdot A \cdot e^{i(\omega t - \alpha)}$$

$$+D \cdot A \cdot e^{i(\omega t - \alpha)} = F_0 \cdot e^{i\omega t}$$

$$-\omega^2 \cdot m \cdot A + i \cdot k \cdot \omega \cdot A + D \cdot A = \frac{F_0 \cdot e^{i\alpha}}{A}$$

$$m\left(\frac{D}{m} - \omega^2\right) + ik \cdot \omega = \frac{F_0 \cdot e^{i\alpha}}{A} \Rightarrow m(\omega_0^2 - \omega^2) + ik \cdot \omega = \frac{F_0}{A}(\cos\alpha + i\sin\alpha)$$

$$m(\omega_0^2 - \omega^2) + ik \cdot \omega = \frac{F_0}{A}(\cos\alpha + i\sin\alpha)$$



Phase shift:

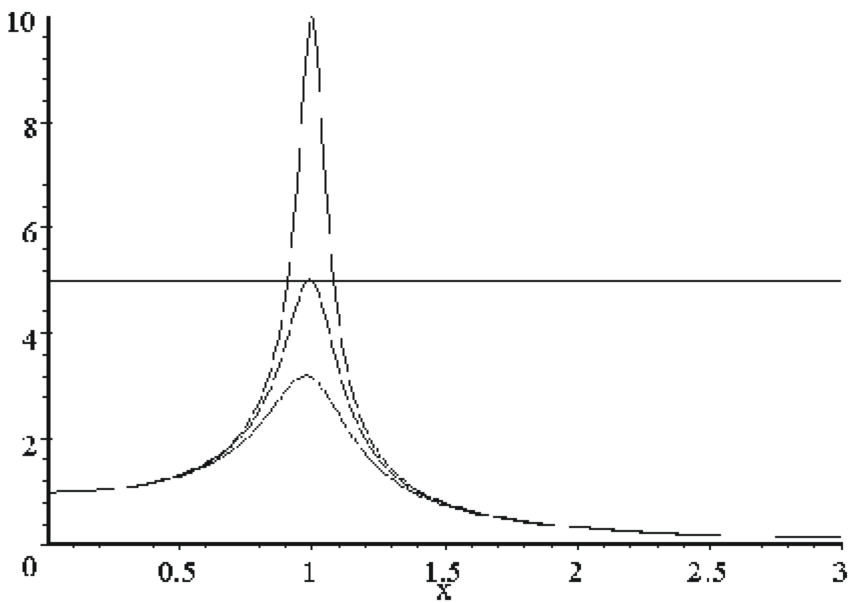
$$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{k \cdot \omega}{m \cdot (\omega_0^2 - \omega^2)} \Rightarrow$$

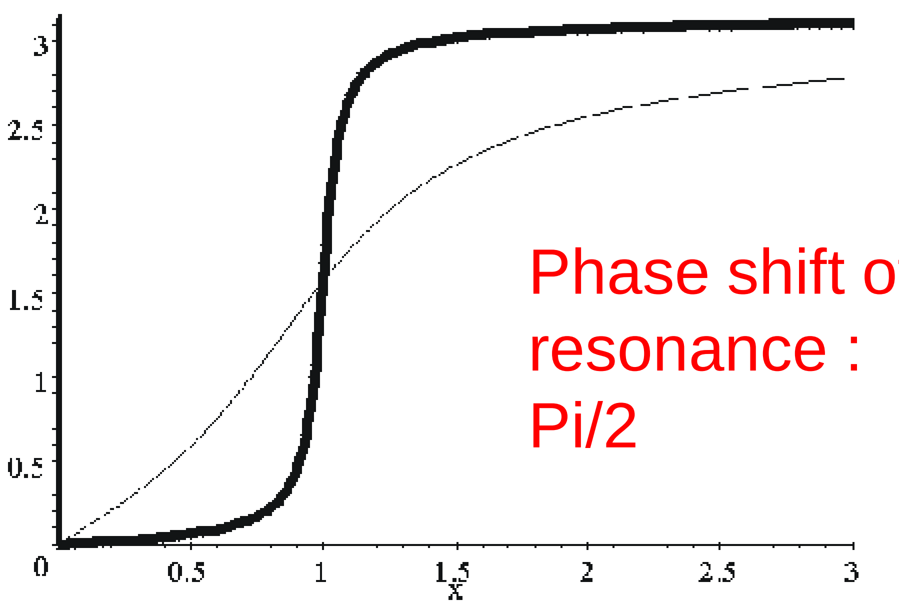
$$\alpha = arc\tan\frac{k \cdot \omega}{m \cdot (\omega_0^2 - \omega^2)}$$

Amplitude: $\left(\frac{F_0}{A}\right)^2 = m^2(\omega_0^2 - \omega^2)^2 + k^2 \cdot \omega^2$

Physical solution: x=A $\cdot \cos(\omega t - \alpha)$

Discussion:





Phase shift of resonance : Pi/2