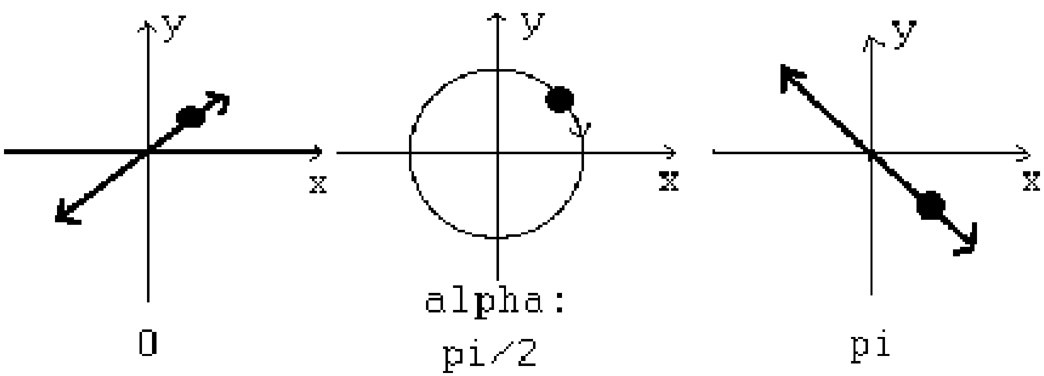
4.3. Superpositions of Ocillations

Here: Superpositions of double harmonic oscillations

a) Same frequency but perpendicular with each other

$$x = x_0 \cdot \cos(\omega t) \qquad y = y_0 \cdot \cos(\omega t + \alpha) \qquad x_0 = y_0$$



All other values produce ellipse

b) Different frequences: ω_1, ω_2 same amplitudes, same direction of oscillations

$$x_1 = x_0 \cdot \cos(\omega_1 \cdot t)$$

$$x_2 = x_0 \cdot \cos(\omega_2 \cdot t)$$

Sum: $x = x_1 + x_2$

$$= x_0 \cdot \cos(\omega_1 \cdot t) + x_0 \cdot \cos(\omega_2 \cdot t)$$

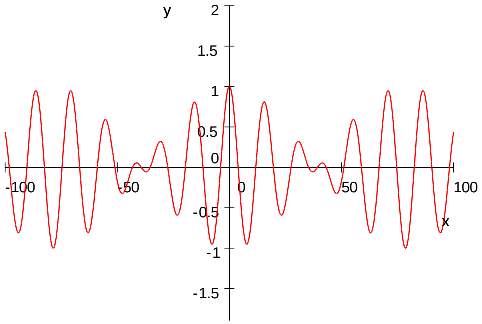
$$= x_0[\cos\omega_1 t + \cos\omega_2 t] = 2 \cdot x_0 \cdot \cos(\frac{\omega_1 - \omega_2}{2} t) \cdot \cos(\frac{\omega_1 + \omega_2}{2} t)$$

Are frequencies only „“little" different

$$\omega \simeq \frac{1}{2}(\omega_1 + \omega_2)$$

$$\Delta\omega = \omega_1 - \omega_2$$

„beat"



$$\Rightarrow x = 2 \cdot x_0 \cdot \cos\left(\frac{\Delta\omega}{2} t\right) \cdot \cos(\omega t)$$

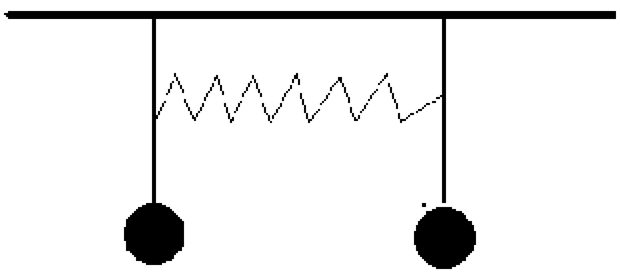
4.4. Coupled oscillators

e.g.: Pendelum

„Small" deflection around the balanced condition

$$\ddot{x} + \omega_0^2 x + \frac{D}{m}(x - y) = 0$$

$$\ddot{y} + \omega_0^2 y + \frac{D}{m}(y - x) = 0$$



Coupling constant D

Approach: $x = Ae^{i\omega t}, \; y = Be^{i\omega t} \Rightarrow$

$$\left(-\omega^2 + \omega_0^2 + \frac{D}{m}\right) \cdot A = \frac{D}{m} \cdot B$$

$$\left(-\omega^2 + \omega_0^2 + \frac{D}{m}\right) \cdot B = \frac{D}{m} \cdot A \Rightarrow$$

Statement of ratio A/B

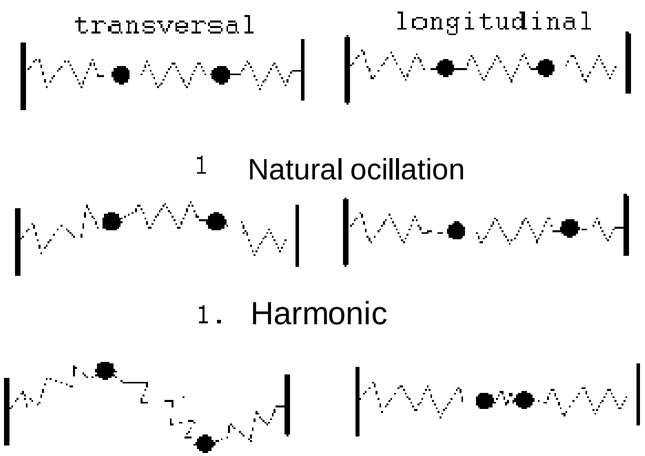
Multiplication $(-\omega^2 + \omega_0^2 + \frac{D}{m})^2 \bullet A \bullet B = (\frac{D}{m})^2 \bullet A \bullet B \Rightarrow$

2 Frequences: $\omega_1^2 = \omega_0^2; \omega_2^2 = \omega_0^2 + \frac{2D}{m}$

Fundamental oscillations: Oscillations in phase/paraphase
General solution: Superposition -→**beat frequency**

Model for a deformable body:

Simple system: 2 balls + 3 springs



n balls with (n+1) springs

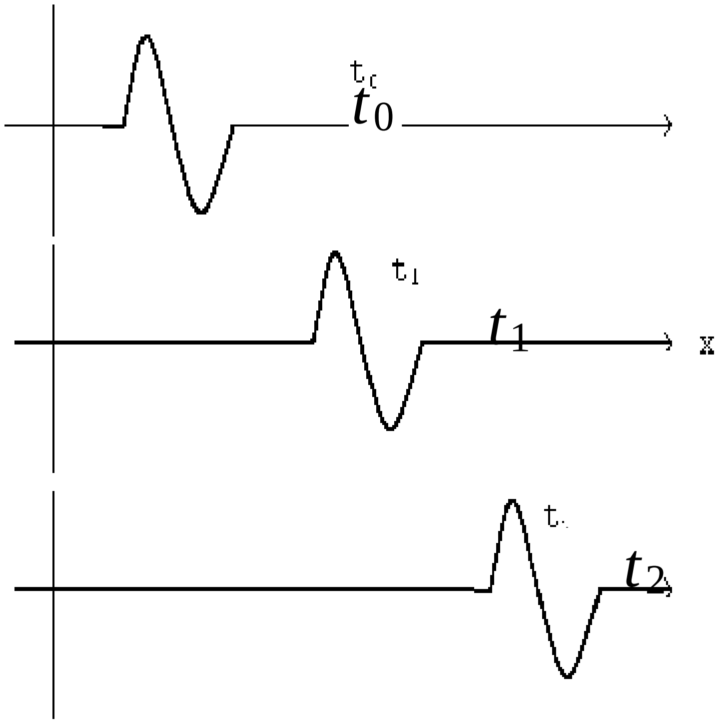
-→

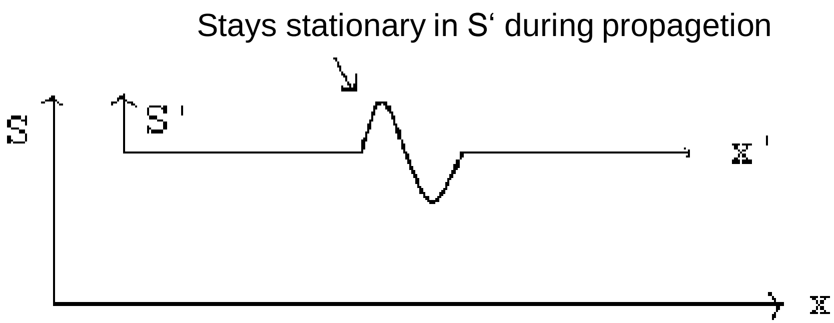
n natural oscillations

4.5. Waves

Waves: Mechanics, Elektromagnetism, Matter, Gravitation

Propagation of a perturbation in a medium: z.B.: Rope



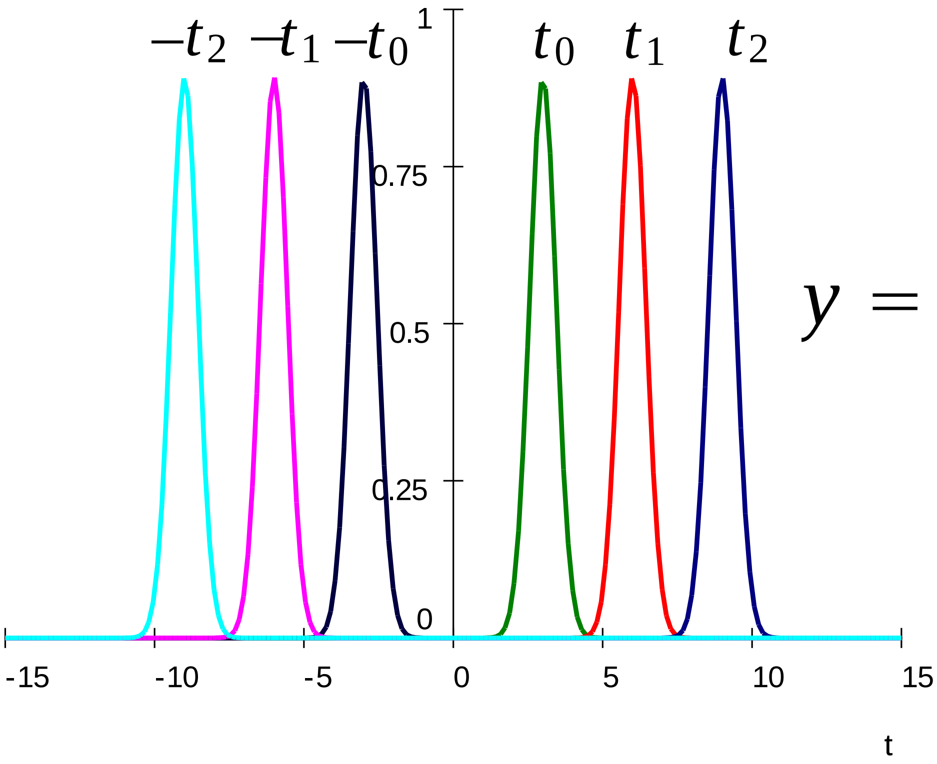


S' moves relatively to S with a velocity v

$x = x' + v \cdot t$ or $x' = x - v \cdot t \Rightarrow$

The propagation spread out towards the right with $y = F(x - v \cdot t)$

or to the left $y = F(x + v \cdot t)$

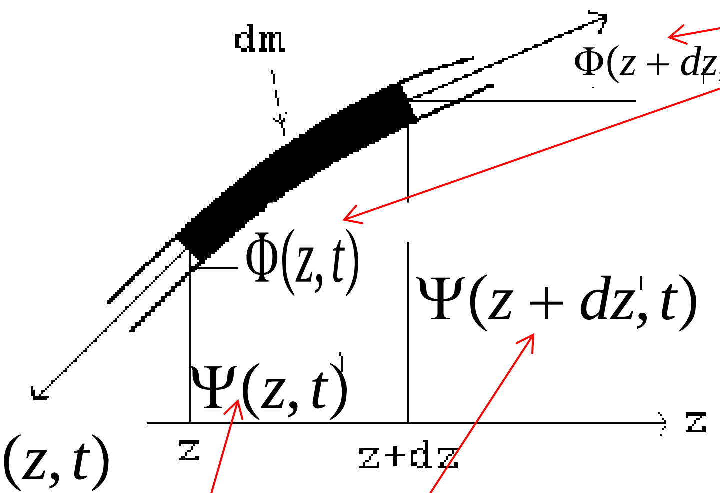


$$y = \frac{1}{\sqrt{2\pi \cdot 1}} \exp\left(-\frac{(x \pm t_i)^2}{1}\right)$$

Description of a transverse oscillation of a rope:

$S_0(z + dz, t)$

angle



$S_0(z,t)$

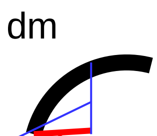
Assumptions:
Gravitation negligible
flat humps of the rope

$$\frac{\partial \Psi(z,t)}{\partial z} \ll 1$$

i.e.: small deflection

Force of rope S_0 alike **state of rest, rope ideal flexible, no friction**

Equations of motions due to Newton for dm:



$$\mu \cdot dz \cdot \frac{\partial^2 \Psi(z,t)}{\partial t^2} = -S_0 \cdot \sin \Phi(z,t) + S_0 \cdot \sin \Phi(z+dz,t)$$

μ : Occupancy of mass: $dm = \mu \cdot dz$

Gradient:
Sin ≈ Tan

given that: $\frac{\partial \Psi(z,t)}{\partial z} \ll 1 \Rightarrow \sin \Phi(z,t) = \frac{\partial \Psi(z,t)}{\partial z}$

Amplitudes

$$\sin\Phi(z+dz,t) = \frac{\partial\Psi(z+dz,t)}{\partial t} \backsimeq \frac{\partial\Psi(z,t)}{\partial z} + \frac{\partial^2\Psi(z,t)}{\partial z^2}\bullet dz$$

Development of Taylor

inserted: $$\boxed{\frac{\partial^2\Psi(z,t)}{\partial t^2} = \frac{S_0}{\mu}\bullet\frac{\partial^2\Psi(z,t)}{\partial z^2}}$$

Equation of an oscillation! transverse polarisation

What is the connection with perturbation on the rope?

General equation of oscillation for funktion y

Diffrentiate: $$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial a}\bullet\frac{\partial a}{\partial t}$$

$$y = F(x - v\bullet t); x - v\bullet t = a$$

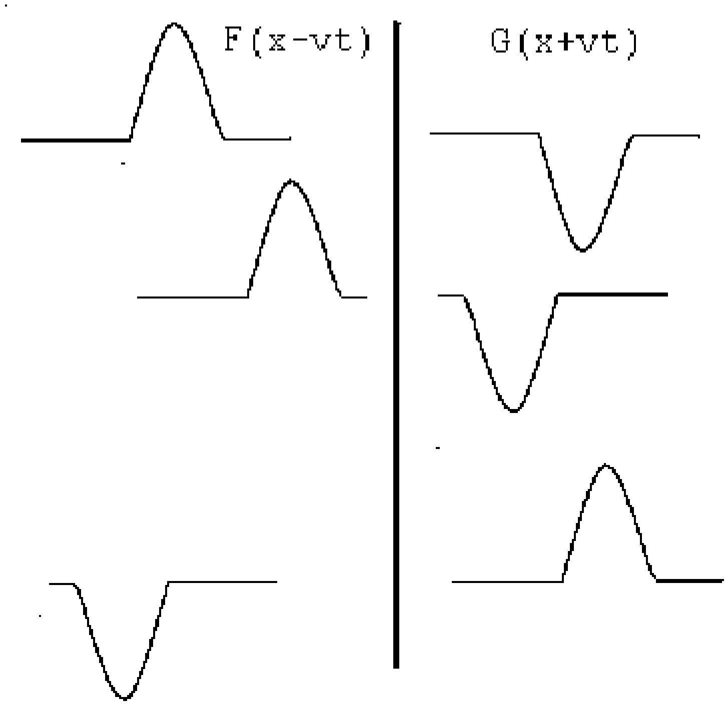
Comparison with wave equation results to:

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 F}{\partial a^2}\left(\frac{\partial a}{\partial t}\right)^2 + \frac{\partial F}{\partial a}\frac{\partial^2 a}{\partial t^2} = \frac{\partial^2 F}{\partial a^2}v^2; \frac{\partial^2 a}{\partial t^2} = 0$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 F}{\partial a^2}\left(\frac{\partial a}{\partial x}\right)^2 + \frac{\partial F}{\partial a}\frac{\partial^2 a}{\partial x^2} = \frac{\partial^2 F}{\partial a^2}$$

$$\frac{\partial^2 F}{\partial a^2}\bullet v^2 = \frac{S_0}{\mu}\frac{\partial^2 F}{\partial a^2} \Longrightarrow v = \sqrt{\frac{S_0}{\mu}}$$

Reflexion of ocillation of a rope:



Fixed wall

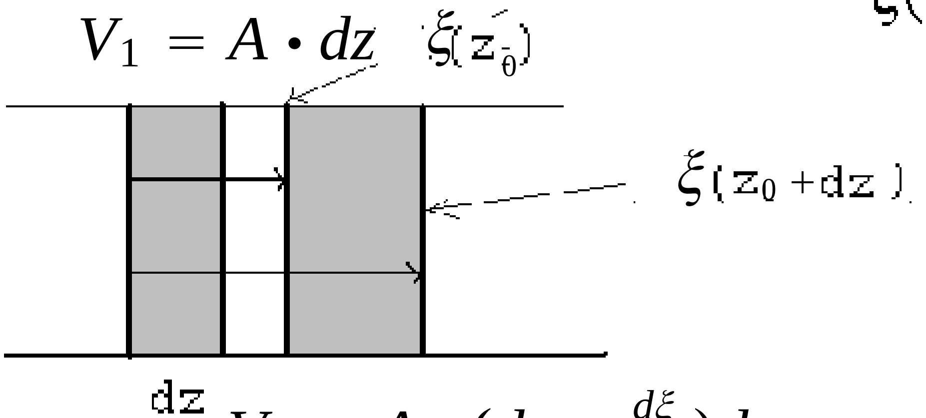
Deflection of rope = 0
at a fixed wall

The coming perturbation gets reflected with negative sign!

Sound waves in gases:

$\xi(z_0)$: Amplitude of oscillations

$$V_1 = A \cdot dz$$



$$V_2 = A \cdot (dz + \frac{d\xi}{dz})dz$$

$$\xi(z_0 + dz) = \xi(z_0) + \frac{\partial \xi}{\partial z} dz$$

front/ back area

Change of volume: $dV = A \frac{\partial \xi}{\partial z} dz \Rightarrow$ change of pressure

$$dp = -p \frac{dV}{V} = -p \frac{\partial \xi}{\partial z}$$

Pressure gradient causes force:

$$dF = -A \cdot dz \cdot \frac{\partial}{\partial z}(dp) = p \cdot A \frac{\partial^2 \xi}{\partial z^2} \cdot dz$$

on the mass:

$$dm = \rho \cdot V = \rho \cdot A \cdot dz$$

Newton: Equations create movement:

$$p \cdot A \frac{\partial^2 \xi}{\partial z^2} = \rho \cdot A \cdot \frac{\partial^2 \xi}{\partial t^2} \Rightarrow \qquad \frac{\partial^2 \xi}{\partial t^2} = \frac{p}{\rho} \frac{\partial^2 \xi}{\partial z^2}$$

Equation of sound waves!