Oscillation / Waves

Frequency of oscillation &Velocity of wave propagation

Starting point: Equation of motion of Newton

e.g:

from

$d^2x/dt^2 + D/m\, x = 0$

from

$$\frac{\partial^2 \Psi(z,t)}{\partial t^2} = \frac{S_0}{\mu} \cdot \frac{\partial^2 \Psi(z,t)}{\partial z^2}$$

$\omega = \sqrt{D/m}$

$v = \sqrt{S/\mu}$

From kinetic energy

From potential energy

$v = \sqrt{p/\rho}$

Examples:

$v = \sqrt{E/\rho}$

E: Module of elasticity

Copper:v=3700m/s

$v = \sqrt{K/\rho}$

K: Module of compression

v(Liquids)=800-1800m/s

Gases: K=p

Air:v=280m/s

But!!

Real:331m/s

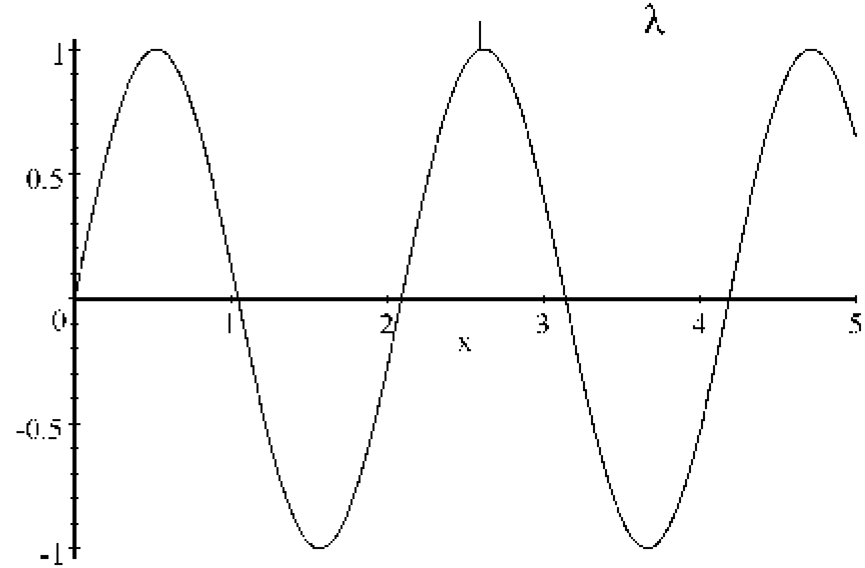
Harmonic waves

Be y = sin $\frac{2\pi}{\lambda}(x - v \cdot t)$

Shortcut:

$$\frac{2\pi}{\lambda} = k; \lambda \ :$$

Next distance with same phases. Each value of phase of wave covers a distance during a second v meter.



In time T: $\lambda = v \cdot T$

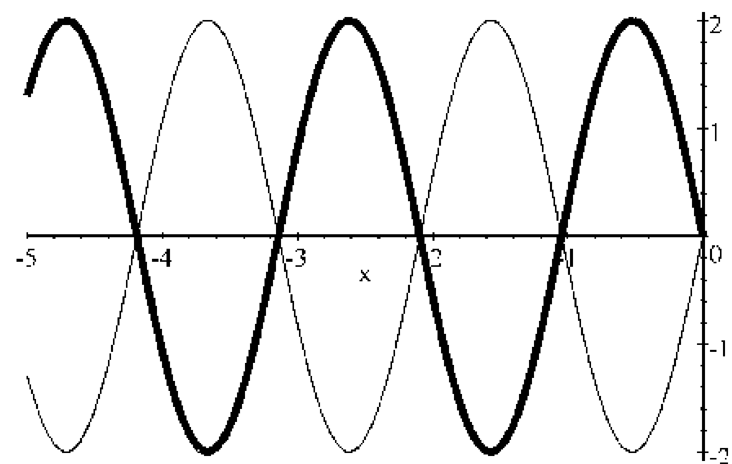
v: velocity of phase

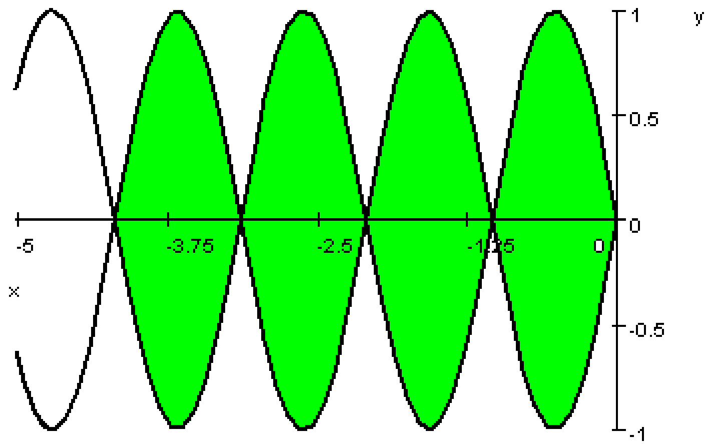
T: Period of oscillation $\omega = 2\pi\nu, \nu = \frac{v}{\lambda}$ or $\omega = v \cdot k \Rightarrow$

harmonic wave:

$$y = \sin(kx - \omega t)$$

Reflexion of harmonic waves: **Standing waves**





Nodes and antinodes

$$y = \sin(kx - \omega t) + \sin(kx + \omega t)$$

$$= \sin kx \cdot \cos \omega t - \cos kx \cdot \sin \omega t$$

$$+ \sin kx \cdot \cos \omega t + \cos kx \cdot \sin \omega t$$

$$= 2 \sin kx \cdot \cos \omega t \Rightarrow$$

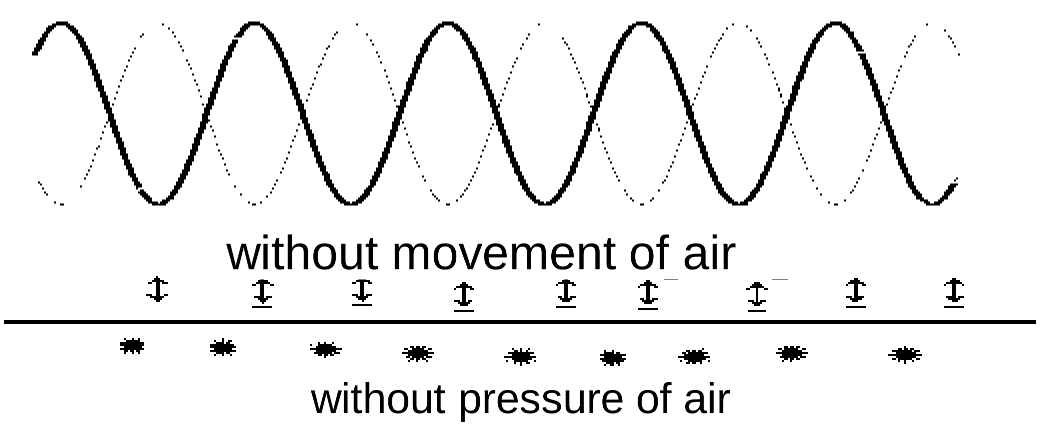
There are times ($\cos \omega t = 0$) where nodes always disappear

There are locations (sinkx=0), where antinodes always disappear

$n \cdot \frac{\lambda}{2}$ With distance from the wall with n=1,2,3

Fixed position in space

e.g.: sound waves:



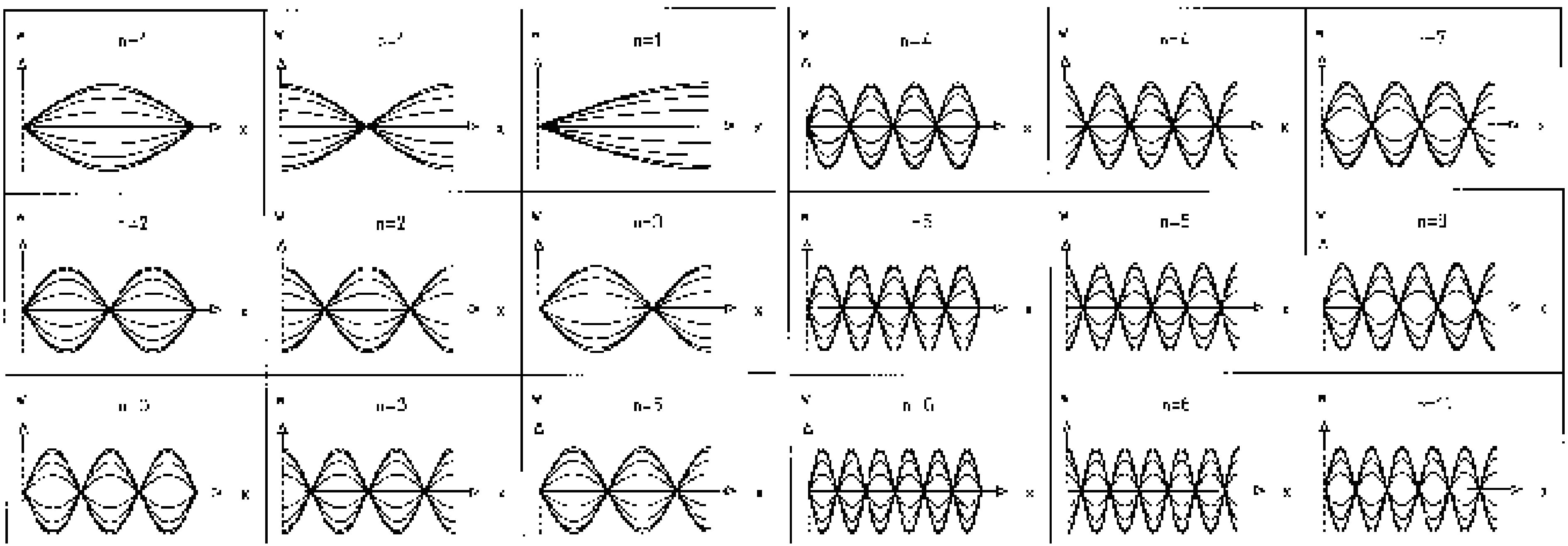
Resonances: Closed/open pipe, with L=(2n+1) $\frac{\lambda}{4}$

Case of resonance: $\nu_n = \frac{(2n+1)v_{Phase}}{4L}$

$$\omega = 2\pi\nu, \nu = \frac{v}{\lambda}$$

open/closed pipe, with L=(n+1) $\frac{\lambda}{2}$

Case of resonance: $\nu_n = \frac{(n+1)v_{Phase}}{2L}$



Rope or string : L=n $\frac{\lambda}{2}; n = 1, 2, 3..$ $\lambda = \frac{2L}{n} \Rightarrow$

only for certain wavelengths or frequencies!

$$\nu = n \cdot \frac{v}{2L} = n \frac{\sqrt{\frac{S_0}{\rho}}}{2L}$$

With S_0 as stringtension

e.g,: violin Bowing of string with the violin bow overtones get excited

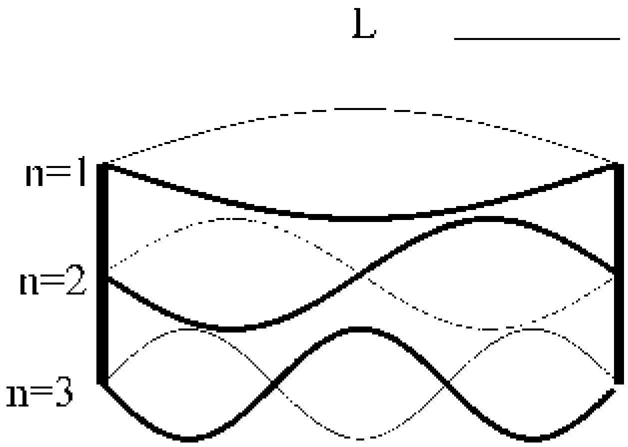
Tone colour w

L: Variation by gripping the fingerboard

Doubling the frequency:1 Oktave

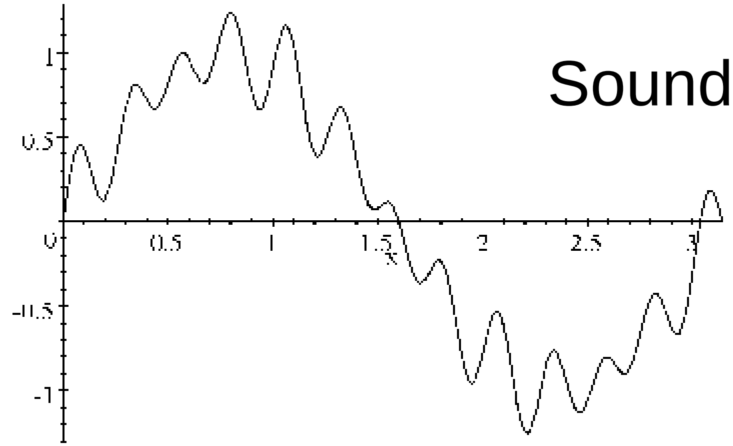
Frequency mixing: Tone colour

Addition of overtones to the key tone



1.-3. Overtones

Generally:



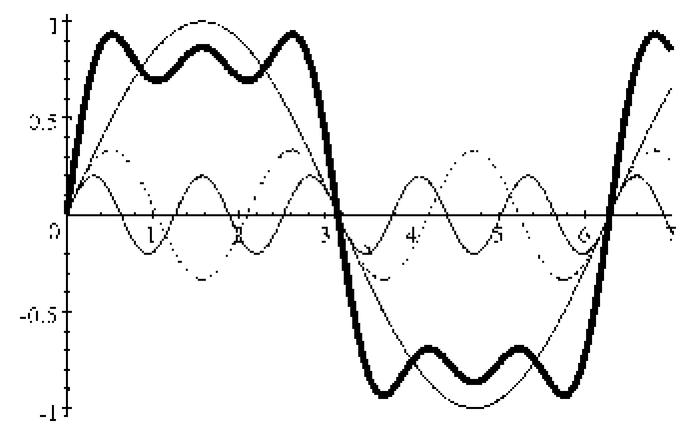
Sound: Rich on overtones!

Quantitative: Frequency analysis, Fourier analysis:

$$\xi(t) = \sum_{n=1}^{\infty} \{\xi_{0,n} \sin(n\omega_1 t + \varphi_n) + i\xi_{0,n} \cos n\omega_1 t\}$$

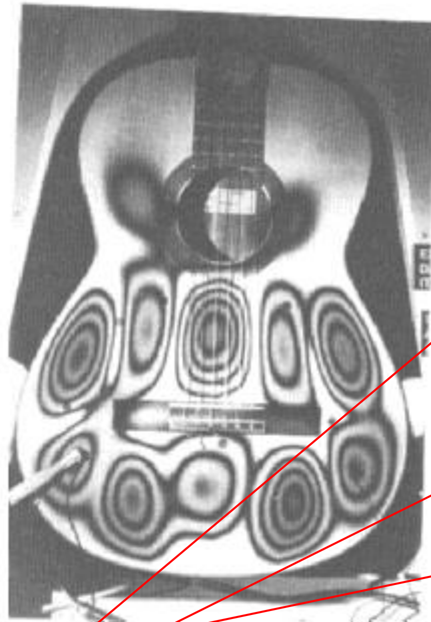
i.e: .Presentation of the amplitude as a linear composition of harmonics!

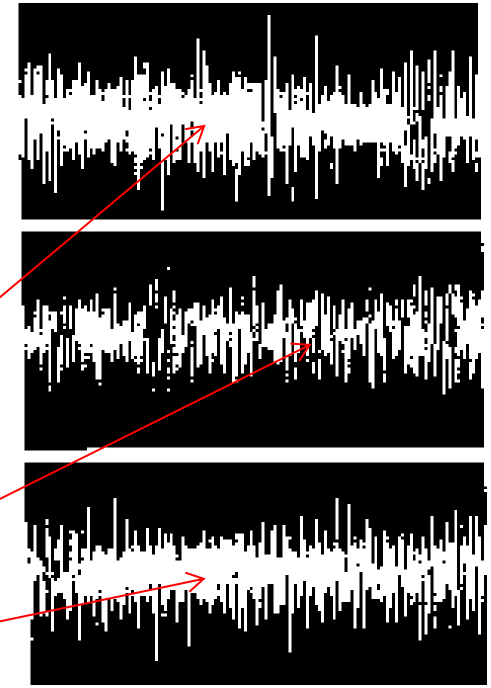
Generally for arbitrary functions of t:
e.g.: Approach of rectangular function by 3 coefficients



How oscillate musical instruments?

How look the signals
in reality ?





How does the human ear
sense all that?

Before beginning of
the lecture

Rehearsal of
an orchstra

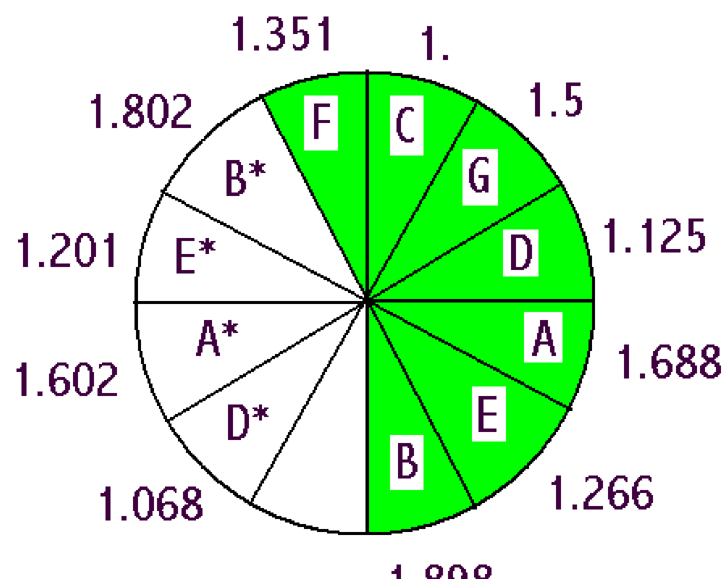
Mendelssohn

Scheme of order of the tones

Starting point!

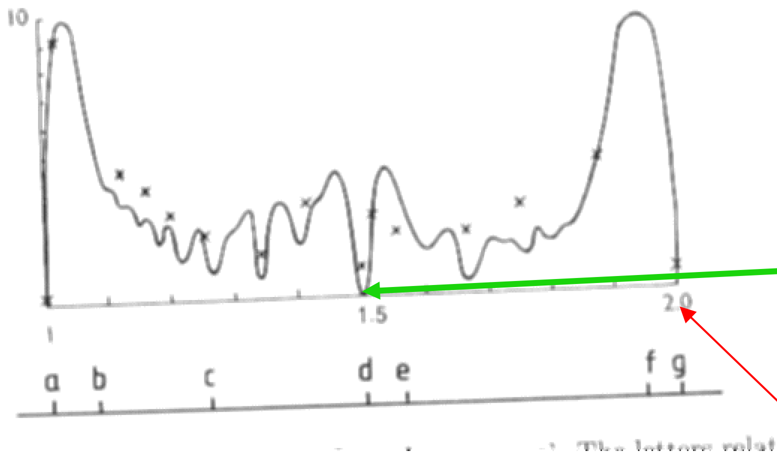
Circle of Pytagoras

$3^n 2^{-m}$



How does the human ear senses all that?

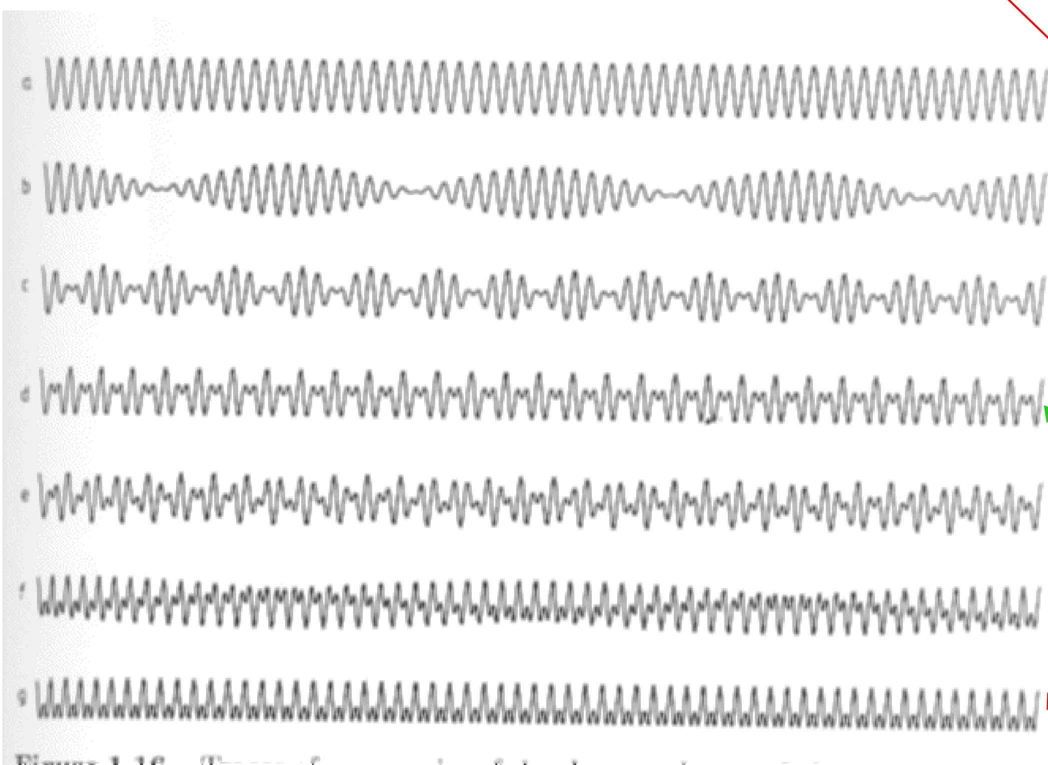
roughness



Due to Helmholtz

Quint

Oktave



Energy of the harmonic elastic wave

$$\xi = \xi_0 \cdot \sin(kx - \omega t))$$

In ΔV with Δm :

$$W = W_{kin} + W_{pot} \qquad W_{kin} = \frac{1}{2}\Delta m \cdot v^2 = \frac{1}{2}\rho \cdot \Delta V \cdot (\frac{d\xi}{dt})^2$$

$$\frac{d\xi}{dt} = -\omega \cdot \xi_0 \cdot \cos(kx - \omega t); \omega \cdot \xi_0 = v_0 \ :$$

Sound particle velocity

$$W_{kin} = \frac{1}{2}\rho \cdot \Delta V \cdot \omega^2 \cdot \xi_0^2 \cdot \cos^2(kx - \omega t)$$

$$W_{pot} = \frac{1}{2}D \cdot \xi^2; \omega = \sqrt{\frac{D}{\Delta m}} \Rightarrow D = \Delta m \cdot \omega^2 = \rho \cdot \Delta V \cdot \omega^2 \Rightarrow$$

$$W_{pot} = \frac{1}{2}\rho \cdot \Delta V \cdot \omega^2 \cdot \xi_0^2 \cdot \sin^2(kx - \omega t)$$

Total energy:

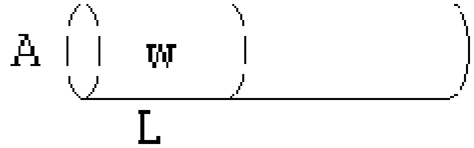
$$W = \frac{1}{2}\rho \cdot \Delta V \cdot \omega^2 \cdot \xi_0^2[\cos^2(kx - \omega t) + \sin^2(kx - \omega t]$$

$$W = \frac{1}{2}\rho \cdot \Delta V \cdot \omega^2 \cdot \xi_0^2$$

Density of energy:

$$w = \frac{W}{\Delta V}$$

Intensity I:

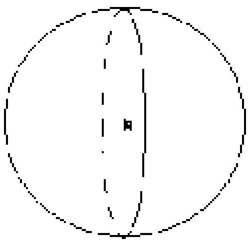


$$L = v_{sound} \cdot \Delta t$$

$$I = \frac{w \cdot V}{A \cdot \Delta t} = \frac{w \cdot v_{sound} \cdot \Delta t \cdot A}{A \cdot \Delta t} = w \cdot v_{sound} \rightarrow I = \frac{1}{2} \rho \cdot \omega^2 \cdot \xi_0^2 \cdot v_{sound}$$

$$[I] = \frac{W}{m^2}$$

Power of a source of sound: P



$$P = \int \vec{I} \cdot \vec{dA}$$

Spherical wave $A = 4\pi R^2 \Rightarrow I = \frac{P}{4\pi R^2}$

Intensity decreases with square of distance!

Examples for power of sources of sound:

	Power (Watt)
speech	10^{-5}
violin	10^{-3}
wind instrument	10^{-1}
loudspeaker	100

Power ratios:

Dezibel (db): $x = 10\log\frac{P_1}{P_2}$

Amplifier and attenuator

Sound intensity:

Threshold of hearing ($\nu = 1000Hz)\ :\ 10^{-12}W/m^2$

Threshold of pain: $10W/m^2$

Definition of sound intensity:

$L_N = 10\log\frac{I}{I_0}$ $\quad I_0 = 2\cdot 10^{-12}W/m^2 \Rightarrow$

absolute Scale

Measure of intensity: Phon

Example: $L_N = 20$ Phon

$\Rightarrow 20 = 10\log\frac{I}{I_0} \Rightarrow$

$\frac{I}{I_0} = 10^2 = 100$

Threshold of pain ca. 130 Phon

Doppler effect

a) Source of sound Q is at rest, observer B moves with velocity v towards B'

C: Velocity of sound

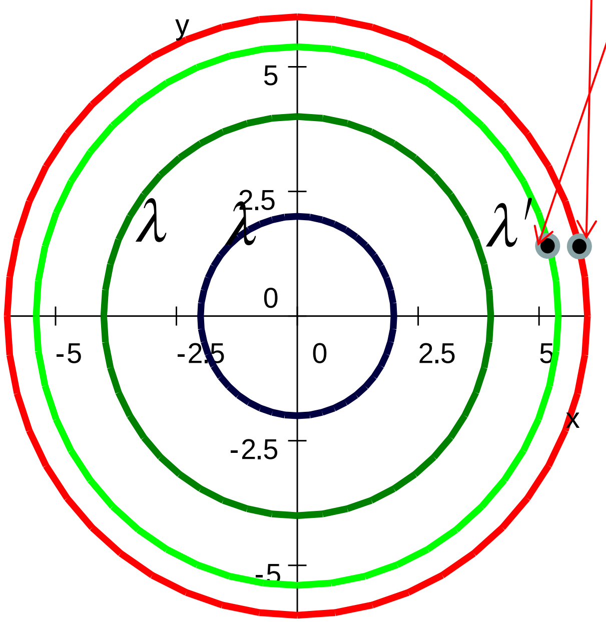
$$\underbrace{c \cdot T'}_{\lambda'} + v \cdot T' = \lambda$$

$$\frac{c}{\nu'} + \frac{v}{\nu'} = \frac{c}{\nu} \Longrightarrow$$

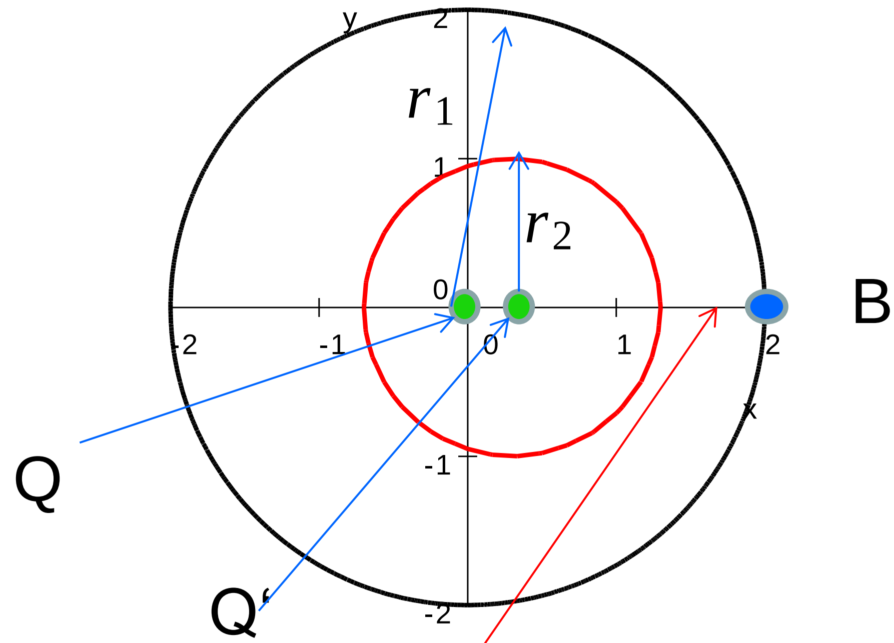
Observer in T‘ from B towards B‘ sees:

$$\nu' = \nu \cdot \left(1 + \frac{v}{c}\right)$$

Observer B, B‘



b) Q moves with v towards observer at rest towards B



with

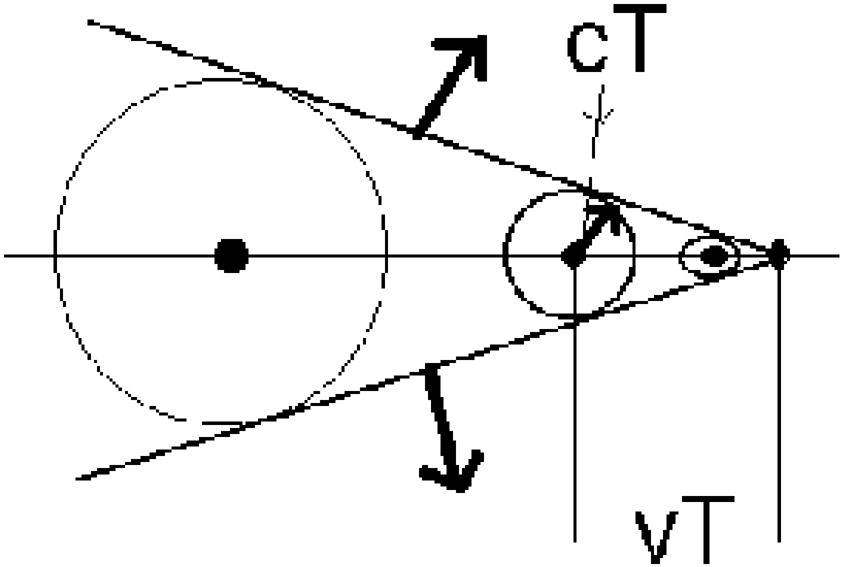
$$\lambda' = r_1 - (r_2 + v \cdot T)$$

$$r_1 - r_2 = cT \qquad = c \cdot T - v \cdot T$$

$$\frac{c}{\nu'} = \frac{c}{\nu} - \frac{v}{\nu} \Rightarrow$$

$$\nu' = \nu\left(\frac{1}{1-\frac{v}{c}}\right)$$

Head waves $v \succ c$



Observation of Mach‘s cone

one can hear a sonic boom!

Quantified: Mach‘s number: $\frac{v}{c}$