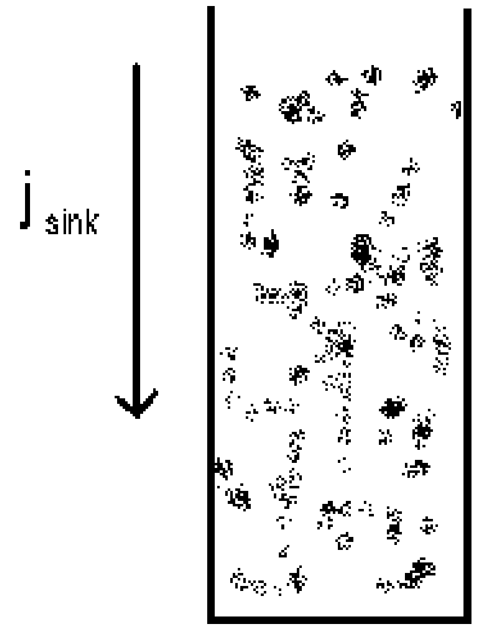
5.14 Transport phenomena

Diffusion, Density distribution



Two effects:

a) Sinking speed

s.above: Stokes (viscous fluid):

$$v = -\frac{m \cdot g}{6\pi\eta \cdot r} \Rightarrow j_{\text{sink}} = n \cdot v = -\frac{m \cdot g \cdot n}{6\pi\eta \cdot r}$$

b)Thermal movement of molecules counteract:

Diffusion

Decisive concentration gradient $\boxed{\frac{dn}{dh}}$

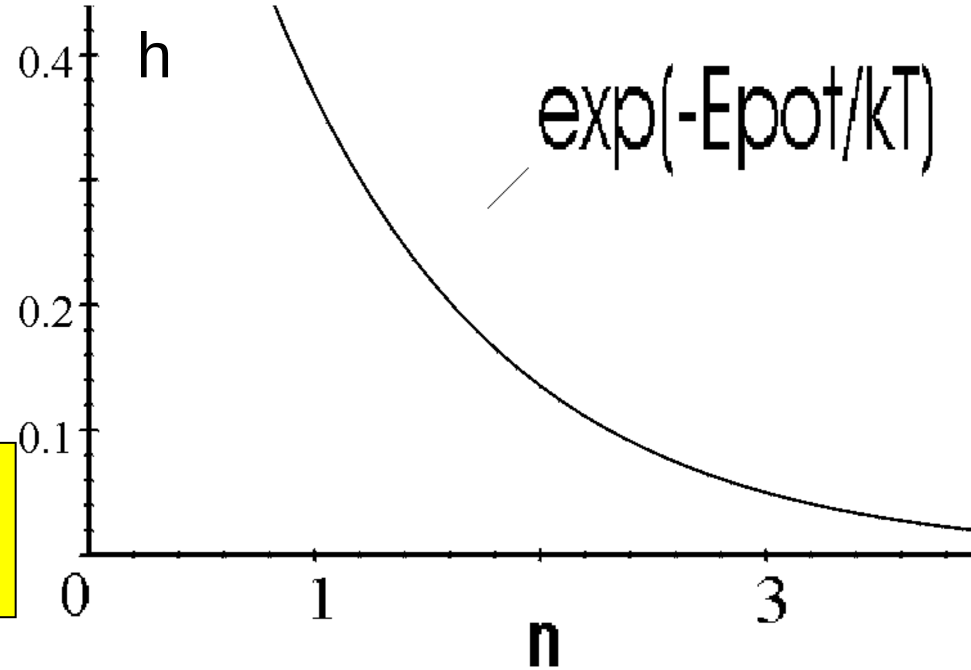
Diffusion current density: $j_{Diff} = -D\frac{dn}{dh}$

Thermal equilibrium: $j = j_{\text{sink}} + j_{Diff} = 0$

$$\frac{-mg \cdot n}{6\pi\eta \cdot r} - D\frac{dn}{dh} = 0; \boxed{\frac{dn}{dh} = \frac{-mg \cdot n}{6\pi\eta \cdot r \cdot D}}$$

or $$\frac{dn}{n} = \frac{-mg}{6\pi\eta\cdot r\cdot D}\,dh$$

$$n(h) = n_0 e^{\frac{-mg}{6\pi\eta\cdot r\cdot D}h}$$



"Barometric formula of altitude"
for density of particles

s.above :
$$n(h) = n_0 e^{-E_{pot}/kT} = n_0 e^{-mgh/kT} \Longrightarrow$$

For gases
$\eta \sim \sqrt{T}$

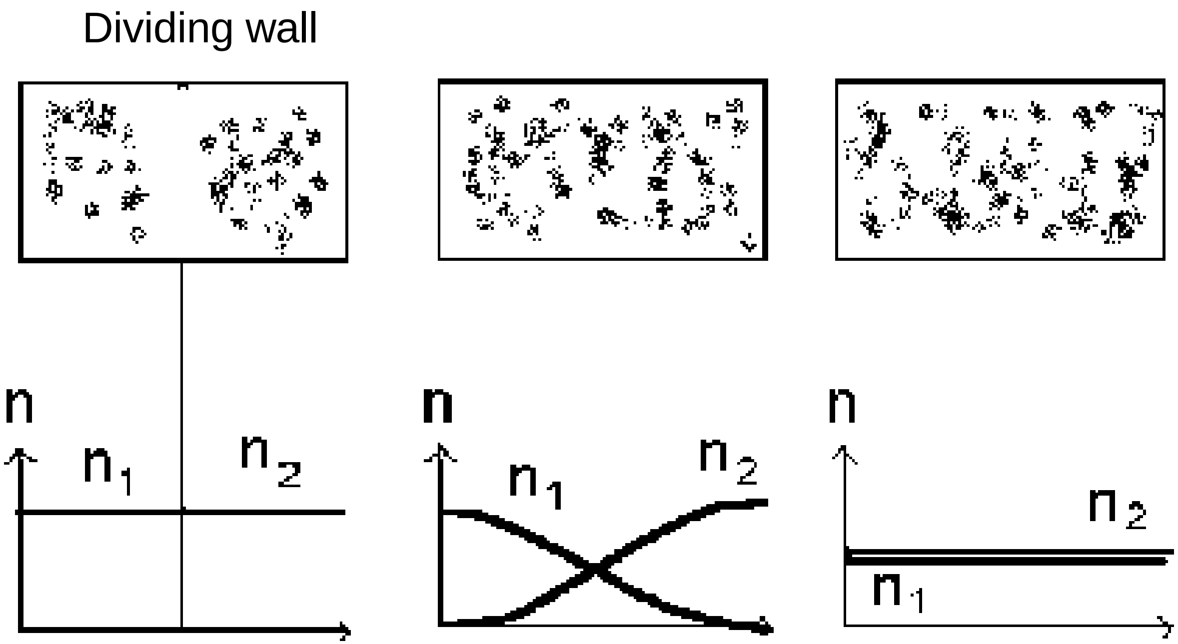
Relation of Einstein: $$D = \frac{kT}{6\pi\eta\cdot r}$$

Diffusion of gases into other ones:

T=const. with

Gas 1	$P_1=n_1kT$
Gas 2	$P_2=n_2kT$

Assumption $P_1=P_2 \rightarrow n_1=n_2$



Thermal movement:
Gases mix,
no additional energy!

Thermal molecular movement ',provides" for mixing: "Diffusion"

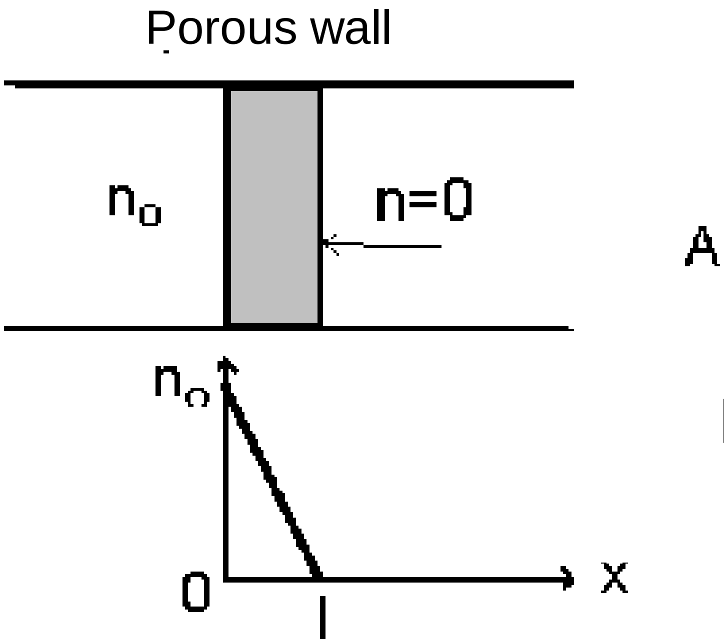
Density of particles i- of gases $\Longrightarrow P_i = n_i \bullet k \bullet T$

Partial pressure

Total pressure P (Sum all partial pressures):

$$\boxed{\sum_i P_i}$$

Daltons rule
Diffusion of gases through porous dividing wall



n and n_0

are not conserved

A : area

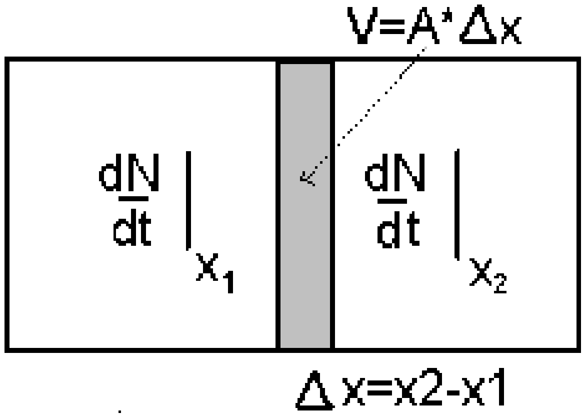
Linear slope: $$\frac{dn}{dx} = -\frac{n_0}{l}$$

continuous molecular current: In $\Delta t \rightarrow \Delta N \quad \Rightarrow \Delta N \sim A \frac{dn}{dx} dt \Rightarrow$

$$i = \frac{\Delta N}{\Delta t} = -D \bullet A \frac{dn}{dx}$$

molecular current

1.Law of Fick
Up until now: Constant slope of concentration



Right/left closed

$$\frac{\partial n}{\partial t} = \frac{1}{V}\left\{ \frac{\partial N}{\partial t}\Big|_{x_1} - \frac{\partial N}{\partial t}\Big|_{x_2} \right\}$$

$\frac{\partial N}{\partial t}\Big|_{x_1}$ Flow 1 into V

$\frac{\partial N}{\partial t}\Big|_{x_2}$ flow out at x2 out of V

$$\frac{\partial n}{\partial t} = \frac{1}{A}\,\frac{\frac{\partial N}{\partial t}\Big|_{x_1} - \frac{\partial N}{\partial t}\Big|_{x_2}}{\Delta x}$$

with 1. law of Fick

$$\frac{\partial N}{\partial t} = -DA\frac{\partial n}{\partial x} \Longrightarrow \frac{\partial n}{\partial t} = -\frac{DA}{A}\underbrace{\frac{\frac{\partial n}{\partial x}\Big|_{x_1} - \frac{\partial n}{\partial x}\Big|_{x_2}}{\Delta x}}_{\frac{\partial^2 n}{\partial x^2}}$$

2. Law of Fick

$$\frac{\partial n}{\partial t} = -D\frac{\partial^2 n}{\partial x^2}$$

Equation of diffusion!

The diffusion constant depends of the mean (thermal) velocity v and the free mean pathlength Λ

Λ : average path of two collitions

$D \sim v_{th} {}^* \Lambda \qquad v_{th} \sim \sqrt{\frac{kT}{m}} \Rightarrow D \sim \frac{\Lambda}{\sqrt{m}}$

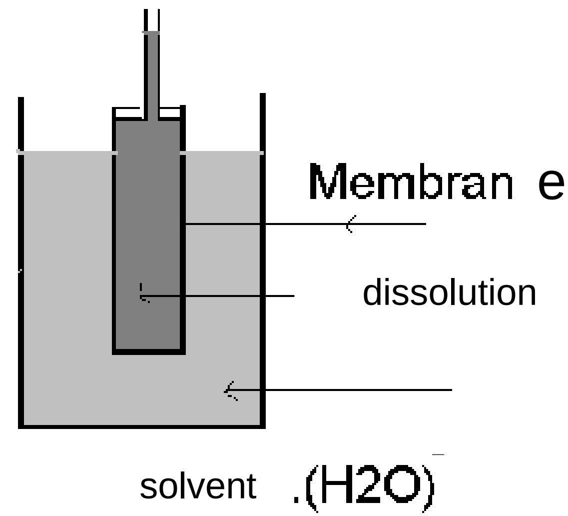
Comparison of two gases: $\frac{D_1}{D_2} = \sqrt{\frac{m_2}{m_1}}$ With the assumption, $\Lambda_1 \backsimeq \Lambda_2$

ligth gases diffuse faster compared to heavy ones!

see exp: Cylinder of clay

Osmosis

Exp.:Pfeffersche cell



e.g.:H2O -molecules diffuse through a semipermeable membrane into treacle
→higher pressure in the cell
→Osmotic pressure

Semipermeable membrane

Law of van t'Hoff: $\boxed{P_{Osmose} \bullet V = M \bullet R \bullet T}$

M:Mass of the material solved in volumen V

experimental result ! C=M/V=Massconcentration

e.g:C=0.1Mol/Liter,T=0° $\Rightarrow P_{Osmose} = 2.24at$

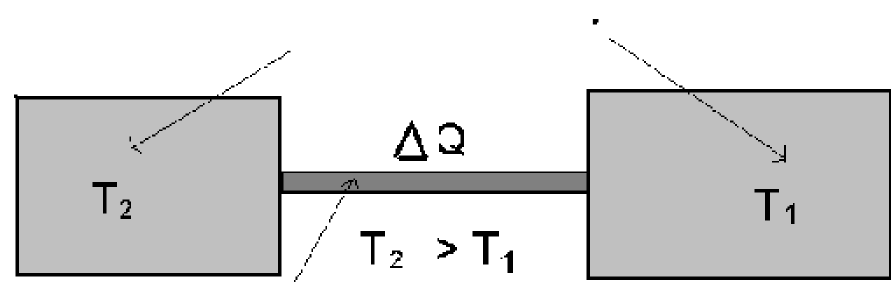
Great consequences in biology: e.g:peas swelling in H_2O

Peas shrink in concentrated saline solution

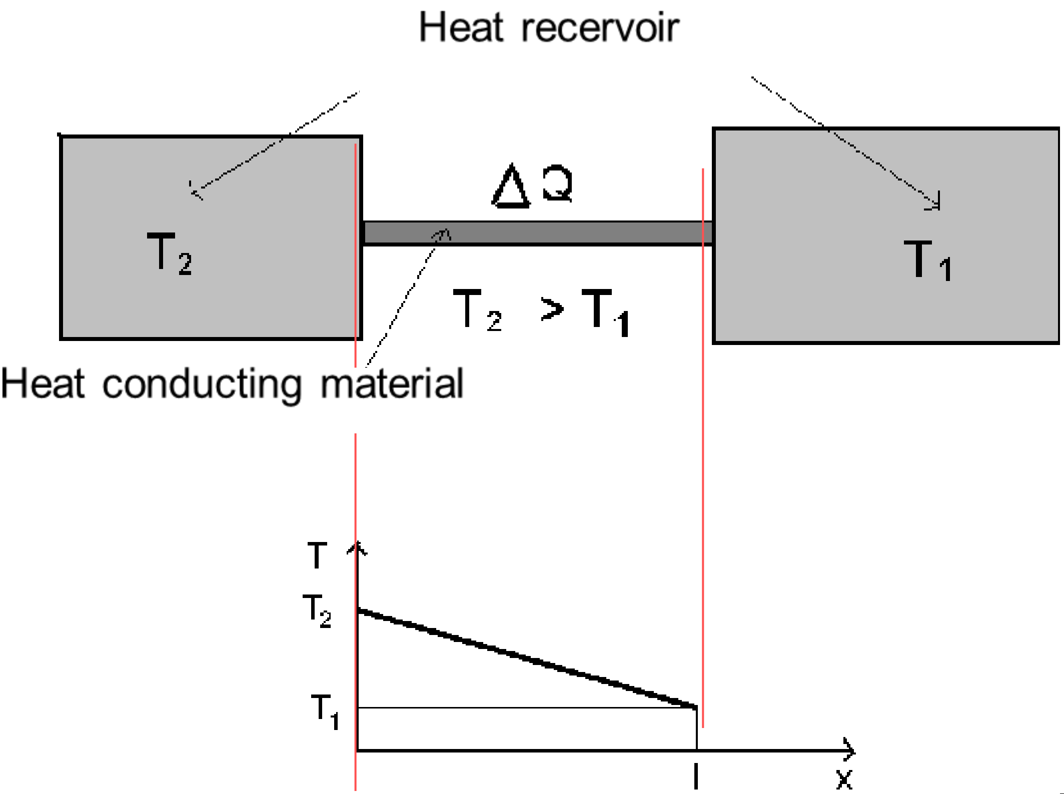
Heat transmission

a) Heat conduction
b) Heat convection
c) Thermal radiation(3.Sem.)

Heat recervoir



Heat conducting material



Temperature gradient $\frac{dT}{dx} = \frac{T_2 - T_1}{l}$

$\Delta Q \sim A \frac{dT}{dx} dt$ or with heat conductivity λ

$$\boxed{\frac{\Delta Q}{\Delta t} = \lambda \cdot A \cdot \frac{T_2 - T_1}{l}}$$

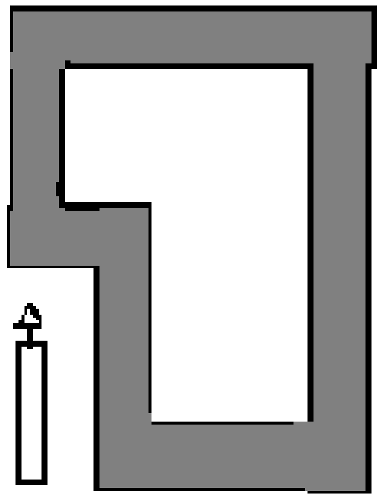
$$[\lambda] = \frac{J}{s \cdot m \cdot k}$$

Examples: λ

Silver	423
Copper	394
Aluminum	201
Iron	71
Porcelain	1
Water	0.6
Air	0.023

Metals: Heat conductivity ~ Electric conductivity

Heat convection



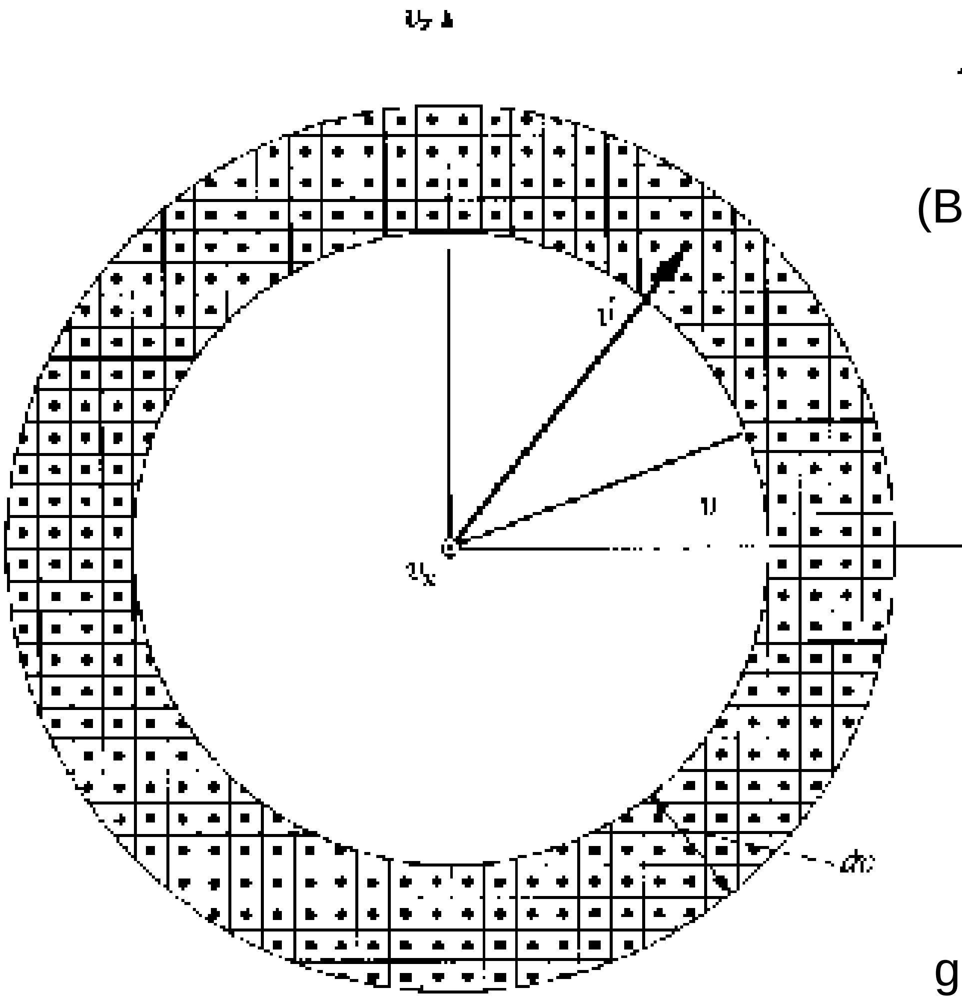
Hot water heating system

Because of variation of density

Addendum: **Velocity distribution due to Maxwell**

Number of particles $dN(v_x, v_y, v_z)$

Volume element of velocity



$$\frac{dN(v_x, v_y, v_z)}{dv_x dv_y dv_z} \sim e^{-\frac{E}{kT}}$$

(Boltzmann-distribution s.above.)

$$dv_x dv_y dv_z = v^2 dv \sin\vartheta d\vartheta d\varphi$$

$$E = \frac{m}{2} v^2 \Rightarrow$$

$$dN(v) = 4\pi v^2 \bullet C \bullet e^{-\frac{m \cdot v^2}{2 \cdot kT}} \bullet dv$$

$$\frac{dN(v)}{dv} = 4\pi v^2 \bullet C \bullet e^{-\frac{m \cdot v^2}{2 \cdot kT}}$$

With C as scale factor
given by the number of particles