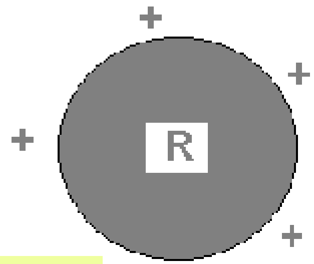
6.6. Capacity



Ball with charge Q against ∞ far away wall:

$$U = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}; U \sim Q$$

$$Q = C \cdot U,$$

with C as capacity

$$[C] = Farad = \frac{C}{V}$$

C depends only from shape of the conductor:

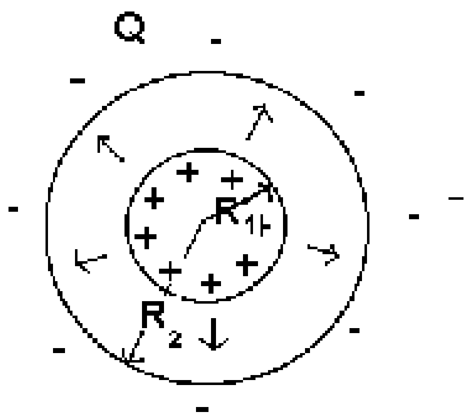
Ball with radius R

$$C = 4\pi\varepsilon_0 \cdot R$$

Capacitor:

At a given voltage:
Seperation of charge depends on C !

Spherical condensor: $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\vec{e}_r$; Potential: $\varphi(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$



Voltage U between both ballcups:

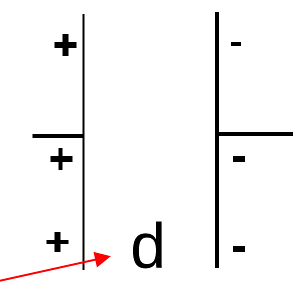
$$U = \varphi_1(R_1) - \varphi_1(R_2) = \frac{1}{4\pi\varepsilon_0}\left\{\frac{Q}{R_1} - \frac{Q}{R_2}\right\}$$

$$= \frac{Q}{4\pi\varepsilon_0}\frac{R_2 - R_1}{R_1 \cdot R_2} = \frac{Q}{C} \quad \text{mit} \quad \frac{1}{C} = \frac{1}{4\pi\varepsilon_0}\frac{R_2 - R_1}{R_1 \cdot R_2}$$

Special case: $R_1 \approx R_2 = R$ and thus $\Delta R \lll R$

$$C = \varepsilon_0 \frac{4\pi R^2}{\Delta R} = \varepsilon_0 \frac{A}{\Delta R}$$

Plate condensor:



U (d)

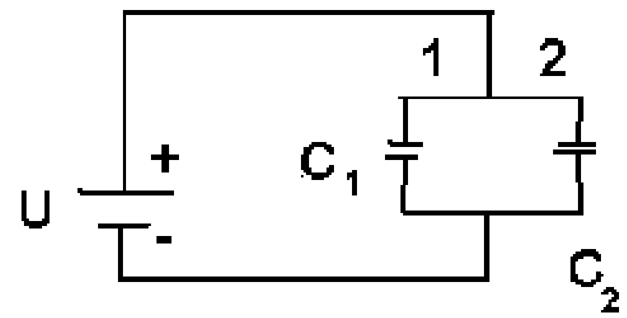
Segment of a spherical condensor with R $\rightarrow \infty$

$\Rightarrow$ $\boxed{C = \varepsilon_0 \frac{A}{d}}$ d distance of the plates

Potential difference : $U = \frac{Q}{C}$

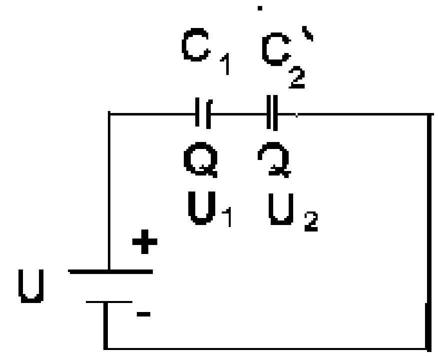
Elektric field: $|\vec{E}| = \frac{d\varphi}{dx} = \frac{\varphi_+ - \varphi_-}{d} = \frac{U}{d} \Rightarrow U = E \cdot d$

Set-up of Condensers



$$Q = Q_1 + Q_2 = C_1 \bullet U + C_2 \bullet U = C \bullet U$$

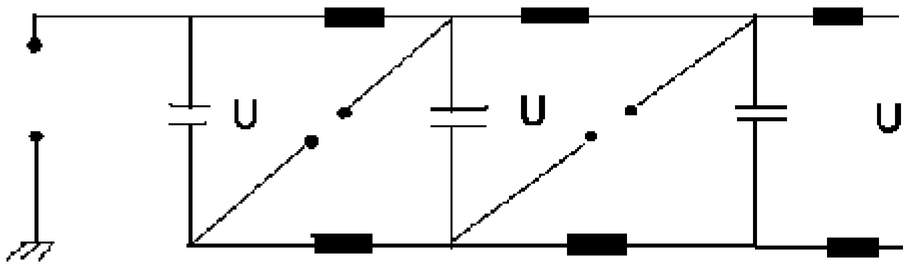
mit $C_1 + C_2 = C$



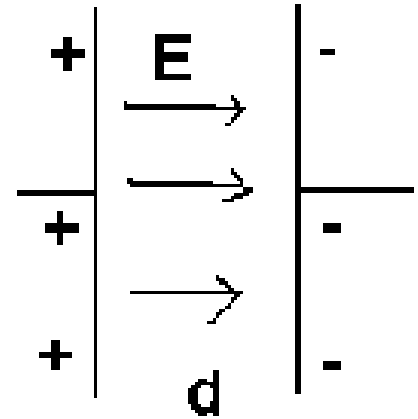
$$U_1 + U_2 = U; \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C}$$

Example: Parallel-serial

Marx-Generator



6.7. Energy within a electric field



$$U = \frac{q}{C};\ E = \frac{U}{d}$$

Continuing charging dq$\Rightarrow$ work necessary:

$$dW = F \cdot d = dq \cdot E \cdot d = U \cdot dq$$

$$W = \int_0^Q U \cdot dq = \int_0^Q \frac{q}{C} dq = \frac{1}{C}\int_0^Q q \cdot dq = \frac{1}{2}\frac{Q^2}{C}$$ or

$$W = \frac{1}{2} C \cdot U_0^2$$

with $U_0 = \frac{Q}{C}$ Example: $U_0 = 1000V$

$$C = \mu F(arad) \quad \boxed{\Rightarrow W = 0.5J = 0.5Ws}$$

1Ws:Wattsecond

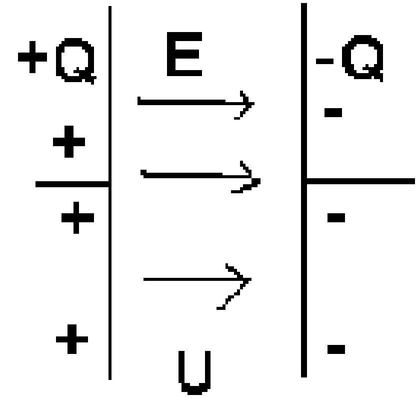
Where is energy situated?

$$W = \frac{1}{2} C \cdot U_0^2 = \frac{1}{2}\varepsilon_0 \frac{A}{d} E_0^2 \cdot d^2 = \frac{1}{2}\varepsilon_0 \cdot E_0^2 \cdot V$$

V: Volume of the field region!

Energy is situated in the electric field

Definition of displacement density:

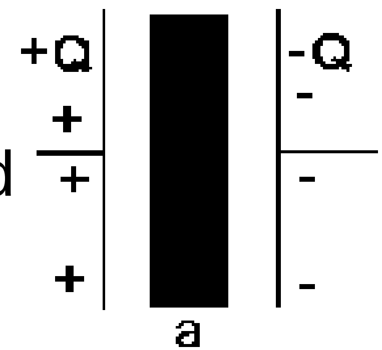


$$Q = C \cdot U = \varepsilon_0 \frac{A}{d} E \cdot d = \varepsilon_0 \cdot A \cdot E$$

Surface density: $\sigma = \frac{Q}{A} = \varepsilon_0 \cdot E = D_0$

$\boxed{\vec{D}_0 = \varepsilon_0 \cdot \vec{E}}$ Displacement density of the vacuum

6.8.Matter in a electric field



Matter: Conductor, distance of the plates: d

$$Q = C_0 \cdot U_0; U_0 = E \cdot d$$

Bringing in of an conductor: Voltage decreases

with conductor: $U = E(d-a) \Rightarrow U = \frac{d-a}{d} \cdot U_0$

because Q= constant $C = \frac{d}{d-a} \cdot C_0$

i.e.: Capacity increases!

Insulator in a electric field!

Glas	ε =5-10
Water	81.1 (18°)
Air	1.000576

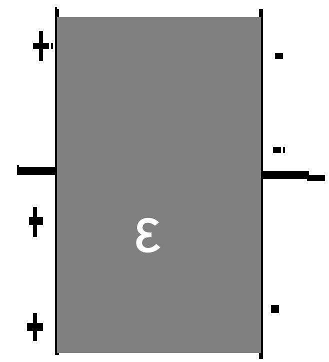
Without dielectric matter: $Q = C_0 \cdot U_0$

with: $U = \frac{Q}{C} = \frac{C_0 \cdot U_0}{C} = \frac{U_0}{\varepsilon}$

Dielectric constant $\boxed{\varepsilon = \frac{C}{C_0}}$

$C = \varepsilon \cdot \varepsilon_0 \cdot \frac{A}{d}$ for plate condensor

How to explain it?



Without dielectricum: $\Phi_0 = E_0 \cdot A = \frac{Q_0}{\varepsilon_0} \; : \; *$

With dielectricum: $U = \frac{U_0}{\varepsilon} \Rightarrow E = \frac{E_0}{\varepsilon}$

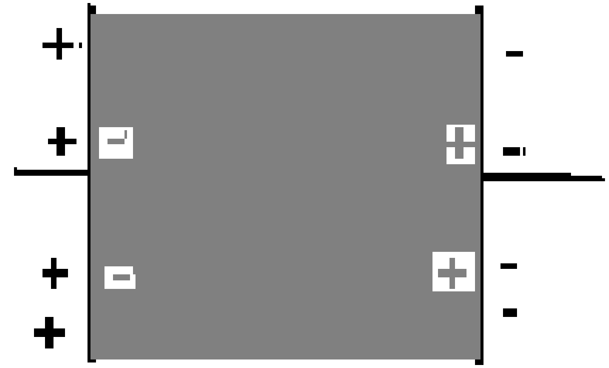
$\Phi = E \cdot A = \frac{Q}{\varepsilon_0} \quad \Phi = \frac{E_0}{\varepsilon} \cdot A = \frac{Q_0}{\varepsilon_0 \cdot \varepsilon} \Rightarrow Q = \frac{Q_0}{\varepsilon}$

On the plates Q_0!

Difference Q_P :

Field lines stop at the surface of the dielectricum

$$Q_P = Q_0 - Q = Q_0\left(1 - \frac{1}{\varepsilon}\right) = \varepsilon_0 \cdot \frac{\varepsilon-1}{\varepsilon} A \cdot E_0;$$



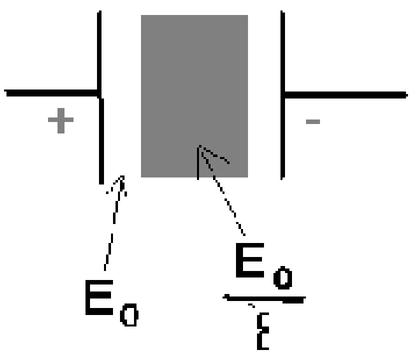
Surface charge density on the dielectricum

$$\sigma_P = \frac{Q_P}{A} = \varepsilon_0 \cdot \frac{\varepsilon-1}{\varepsilon} \cdot E_0 = \varepsilon_0 \cdot (\varepsilon - 1) \cdot E$$

Elektric dipolmoment of the isolator:

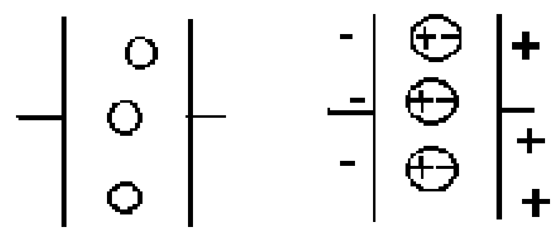
$$\vec{P} = Q_P \cdot \vec{d} = \sigma_P \cdot A \cdot \vec{d} = \varepsilon_0 \cdot (\varepsilon - 1) \cdot V \cdot \vec{E}$$

Polarisation: $\vec{p} = \frac{\vec{P}}{V}$ $\quad \vec{p} = \varepsilon_0 \cdot (\varepsilon - 1) \cdot \vec{E} = \varepsilon \cdot \varepsilon_0 \cdot \vec{E} - \varepsilon_0 \cdot \vec{E}$



Polarisation:

a) Molecules without permanent dipol-moment



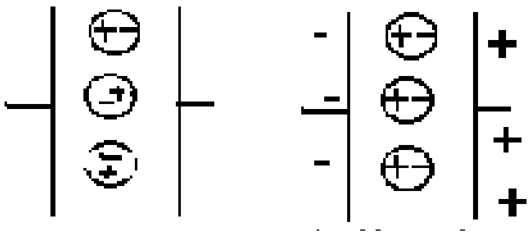
Without field

With field

Electric dipolmoments get induced!

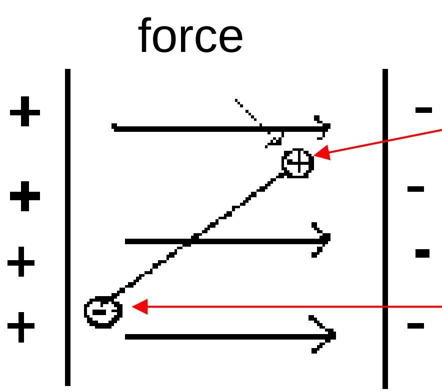
',Displacement polarisation"

b) Molecules with a permanent dipolemoment



without field   partial polarised

"Orientation polarisation"



force: $\vec{F} = Q \cdot \vec{E};$

l: distance of charges

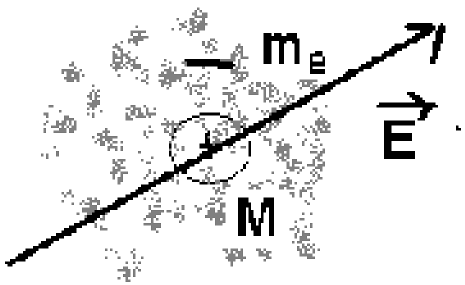
$$\vec{F}' = -Q \cdot \vec{E};$$

$\Longrightarrow$ torque : $\vec{T} = \vec{l} \times \vec{F} = Q \cdot \vec{l} \times \vec{E}$

with : $Q \cdot \vec{l} = \vec{P}$: electric dipolemoment

$$\boxed{\vec{T} = \vec{P} \times \vec{E}}$$

Polarisibility of atoms in electric alternating fields :



$\vec{E}$: outer field, masses M, m_e with M >>m_e

the electron gets moved by force out of balance

$$m_e \frac{d^2x}{dt^2} + m_e \cdot \omega_0^2 \cdot x = q \cdot E_x^0 \cdot \cos\omega t$$

solution with x=x_0 *cosω t

$$\omega^2 \cdot m_e \cdot x_0 \cdot \cos\omega t + m_e \cdot \omega_0^2 \cdot x_0 \cdot \cos\omega t = q \cdot E_x^0 \cdot \cos\omega t$$

$$x_0 = \frac{q \cdot E_x^0}{m_2(\omega_0^2 - \omega^2)}$$

The deflection corresponds to an oscillating dipolemoment

$$p_x = q \cdot x$$

m_e=m_2

$$\Rightarrow p_x = \frac{q^2 \cdot E_x}{m_2(\omega_0^2 - \omega^2)} = \varepsilon_0 \cdot \alpha(\omega) \cdot E_x$$