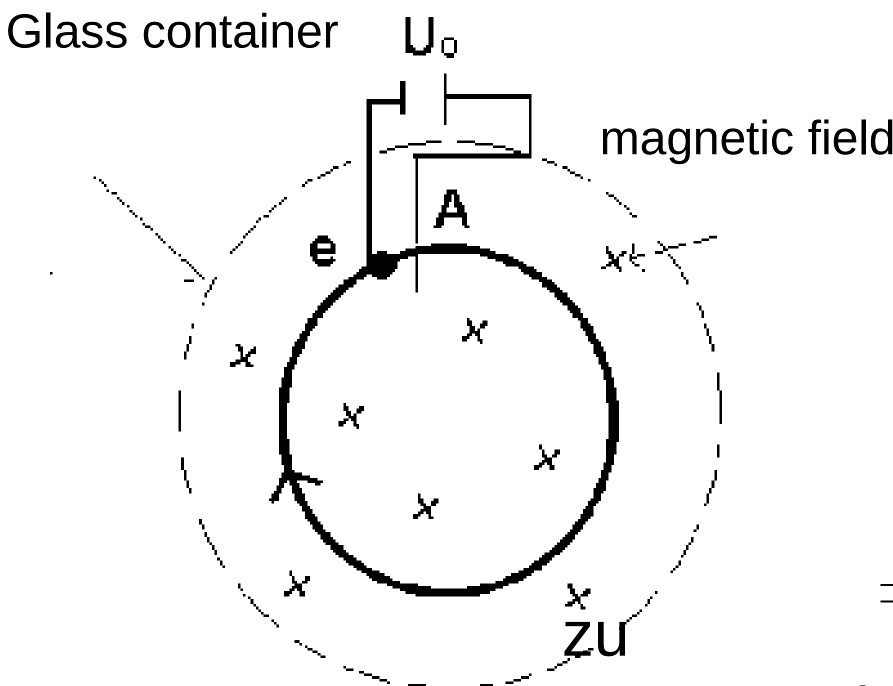
8.5. Movement of charged particles in a magnetic field



Cathode: Thermionic emission of e, acceleration towards cathode

Kinetic energy:

$$E_{kin} = e \bullet U_0 = \frac{1}{2} m v^2$$

$$\Rightarrow v = \sqrt{2 \frac{e}{m} U_0}\,, \quad \frac{v^2}{r} = \omega = \frac{e}{m} B$$

Cyclotron frequency

B ⊥ on area level

Lorentz-force : $|\vec{v}|$ Not changed by $\vec{B}$

Acceleration ⊥ to $\vec{v}$ Constant in time → $\frac{K}{m} = \frac{e \cdot v \cdot B}{m} = \frac{v^2}{r}$

$|\vec{v}|$ =2π·r·v und 2π·v=ω

from $\frac{v \cdot \omega \cdot r}{r} = \frac{e \cdot v \cdot B}{m}$

$$\boxed{\frac{v^2}{r} = \omega = \frac{e}{m} \cdot B \quad \text{Cyclotron frequency}}$$

from a measurement of ω und B → $\frac{e}{m} = -1.75881962 \times 10^{11}\ \mathrm{C\ kg^{-1}}$

Orbit in a homogeneous field B with $v_\perp$ and $v_\parallel$ towards B:

spiral

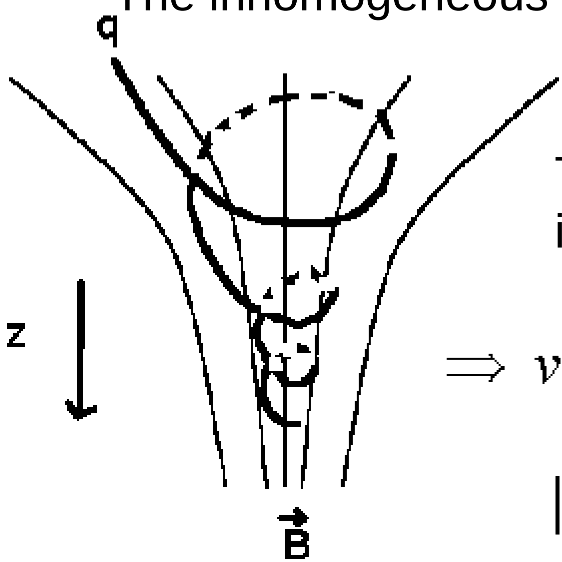
$v_\parallel$ no change!

$$\frac{v_\perp}{r} = \omega = \frac{e}{m} B \Rightarrow$$

helical movement: Orbit in a homogeneous field B is given by equation of force:

$$\vec{K} = q \cdot (\vec{v} \times \vec{B})$$

The inhomogeneous case



The Lorentz force has a component into -z-direction →Particles get reflected

$\Rightarrow v_z$ Gets smaller by entering into inhomogeneous field ab,

$|\vec{v}|$ remains unchanged →$v_\perp$ has to increase.

Angular momentum around the vertical axis of symmetry remains constant.

Because the direction of force is always towards the axis of symmetry →

ω ~ to local field B→

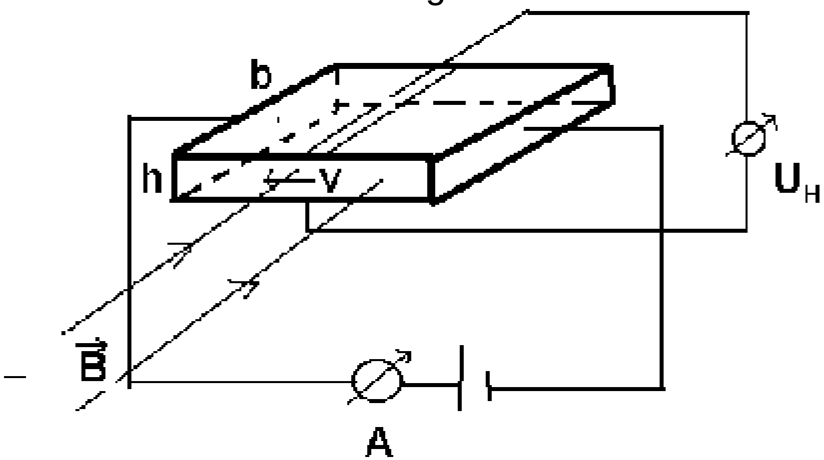
$$m \cdot v_\perp \cdot r = m \cdot \omega \cdot r^2 = \text{constant}$$

$$\underbrace{B \cdot r^2}_{} = const.$$

revolved magnet flux remains constant

8.6. Der "Hall-Effekt"

electrons in a magnetic field



Ass.: Carriers of charge are electrons with current v-

$$\vec{F}_B = -e(\vec{v}_- \times \vec{B})$$

electrons get upwards deflected

Hall voltage inside up and down

buildup of a field $\vec{E}$

is origin of force:

$$\vec{F}_E = -e \cdot \vec{E} \qquad E = \frac{U_H}{h}$$

equilibrium:

density

$$e \cdot E = e \cdot v_- \cdot B \Rightarrow$$

$$I = -e \cdot n_- \cdot A \cdot v_-; A = b \cdot h$$

$$e \cdot v_- = -\frac{I}{n_- \cdot b \cdot h} \Rightarrow e\frac{U_H}{h} = -\frac{I \cdot B}{n_- \cdot b \cdot h} \Rightarrow$$

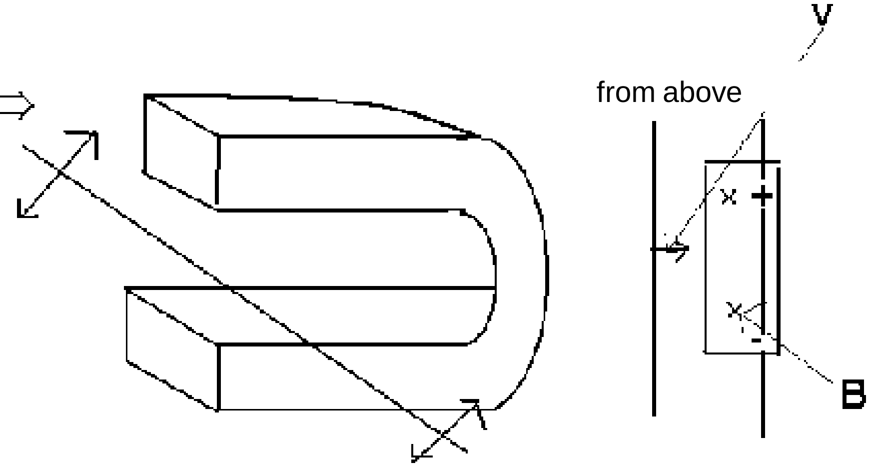
$$U_H = -\frac{1}{e \cdot n_-}\frac{I \cdot B}{b}; -\frac{1}{e \cdot n_-}$$

Hall-constant

Hall probe: Measurement of B-fields

8.7 Moving metallic conductors

$$q \cdot E + q \cdot v \cdot B = 0 \Rightarrow E = -v \cdot B \Rightarrow$$

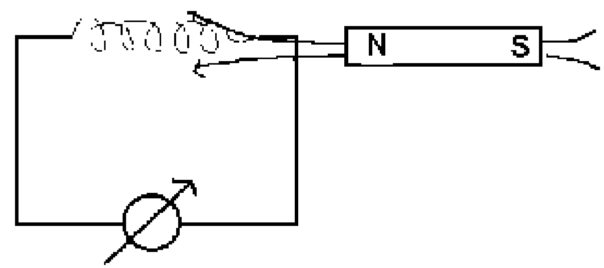


EMK ε within a wire

$$\varepsilon = \int_0^l \vec{E} \cdot d\vec{r} = -v \cdot B \cdot l$$

Basic principle of generators of voltage:
By moving of conductors in a static magnetic field

Reversal: Moving-coil galvanometer

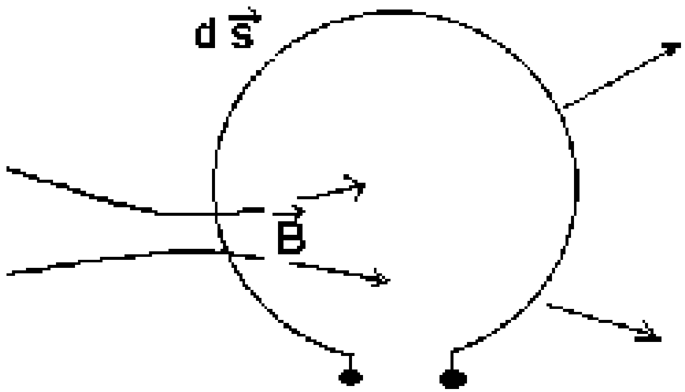


8.8.Electromagnetic Induction

Observation by Faraday

As soon the magnetic flux Φ within
a cross-sectional area of the coil changes
→ a EMK emerges

$$\boxed{\varepsilon = -\frac{d\Phi}{dt}; \Phi = \vec{B} \cdot \vec{A}}$$



$$\frac{d}{dt}\Phi = -\frac{d}{dt}\int \vec{B} \bullet d\vec{A}$$

$$U_{ind} = \int_1^2 \vec{E} \bullet d\vec{s} \Rightarrow$$

$$\int \vec{E} \bullet d\vec{s} = -\frac{d}{dt}\int \vec{B} \bullet d\vec{A}$$

Law of Faraday

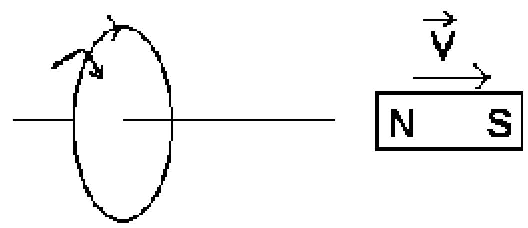
Rule of Lenz:

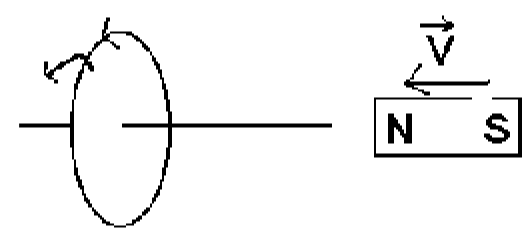
$$I = \frac{U_{ind}}{R} = -\frac{\dot{\Phi}}{R}$$

R= resistor of current loop

Direction of I : from energy conservation:

I creates because of R heat





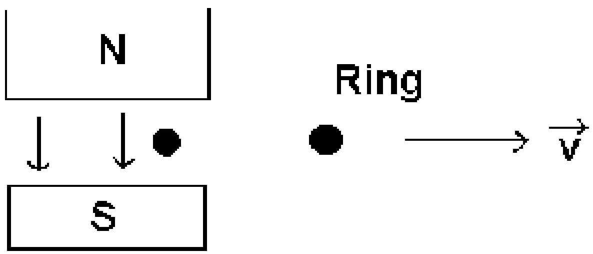
and therefore has to be determined by the movement of bar magnet.

→ Bar magnet gets slowed down → Direction of current

Rule of Lenz:

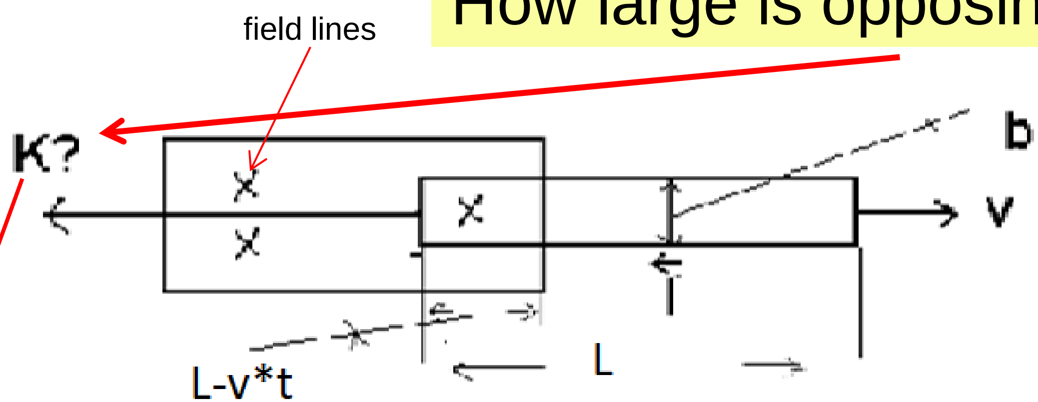
The induced current has always a direction in order to oppose the change of flux which he created.

Example:



Current gets induced,
In order to keep flux constant!

How large is opposing force



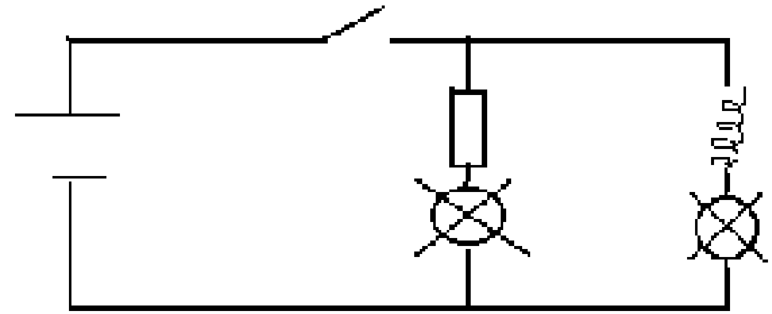
$$v = \frac{dx}{dt}$$

$$I = \frac{U_{ind}}{R} = \frac{-\dot{\Phi}}{R} = \frac{B \cdot b \cdot v}{R}; \Phi = B \cdot b(l - v \cdot t)$$

$$\boxed{\vec{K} = b(\vec{I} \times \vec{B}) = \frac{-B^2 \cdot b^2}{R} \vec{v}}$$

The breaking force ~ velocity of the bar

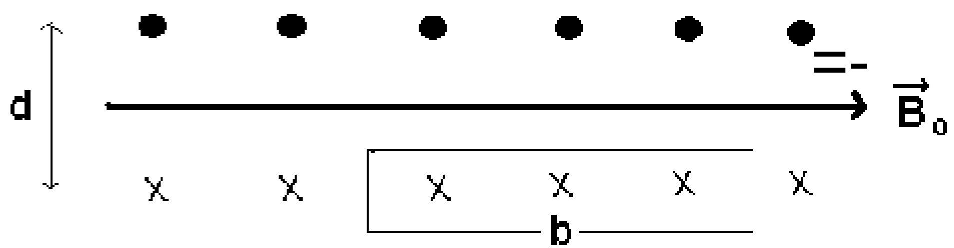
8.9. Self-induction



$$\Phi \sim I \Rightarrow U_{ind} = -\frac{d\Phi}{dt} = -L\frac{dI}{dt}; L:$$ Inductivity

$$[L] = \frac{V \cdot s}{A} = H(enry)$$

e.g.: L of "long" coil



$$B_{outside} \lll B_0$$

Law of Ampere:

$$B_0 \cdot b - B_{outside} \cdot b = B_0 \cdot b$$

in rectangle of integration

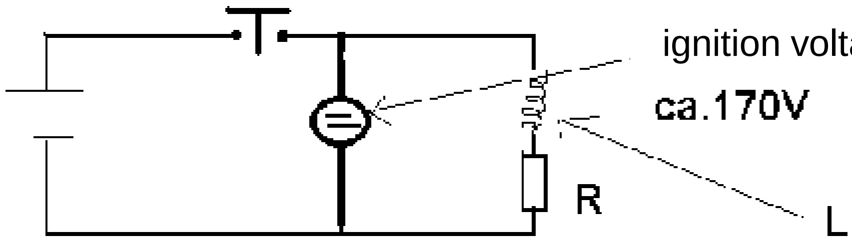
$$B_0 \cdot b = \mu_0 \cdot N \cdot I; N:$$ turns

$$\Rightarrow B_0 = \mu_0 \cdot n \cdot I; n = \frac{N}{b}$$ For the whole coil:

$$\Phi = B_0 \cdot A = \mu_0 \frac{A \cdot N}{l} I \Rightarrow U_{ind} = -N\frac{d\Phi}{dt}; \frac{d\Phi}{dt}$$ in one turn

$$= -\frac{\mu_0 \cdot A \cdot N^2}{l} \cdot \frac{dI}{dt}; \frac{\mu_0 \cdot A \cdot N^2}{l} = L$$

Exp: Ignition of an electric discharge lamp



At opening of T (push button) high voltage:

short lightning

Henry

e.g:R=2 $\Omega, I = \frac{U_0}{R} \simeq 2A$ L=630H $\Delta t \simeq 0.1s \Rightarrow$

$$\boxed{U_{ind} = -L \cdot \dot{I} = -L \frac{\Delta I}{\Delta t} \simeq -630 \frac{-2}{0.1} = 12600.0V}$$

$$1H = 1 \frac{V \cdot s}{A} = \Omega \cdot s$$

Application: Ignition in a car