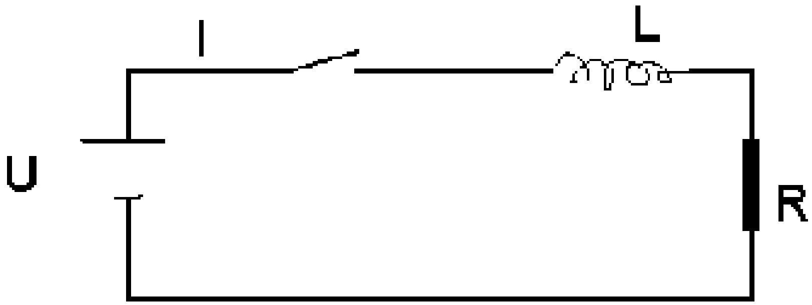
Current increase/ drop of a circuit with L and R



Summe aller Spannungen=0

$$U - I \bullet R - L \bullet \frac{dI}{dt} = 0$$

or $$\frac{dI}{dt} + \frac{R}{L}\left(I - \frac{U}{R}\right) = 0 \; *$$

with $I - \frac{U}{R} = x \Rightarrow \frac{dx}{dt} = \frac{dI}{dt}$ or for $$\frac{dx}{dt} + \frac{R}{L}x = 0$$

or: $\frac{dx}{x} = -\frac{R}{L}dt \Rightarrow \ln x = -\frac{R}{L}t + \ln A($ constant of integration)

$\Rightarrow x = A \bullet e^{-\frac{R}{L}t} \Rightarrow I = \frac{U}{R} + A \bullet e^{-\frac{R}{L}t}$ for t=0 ', switch on'', $I = 0$

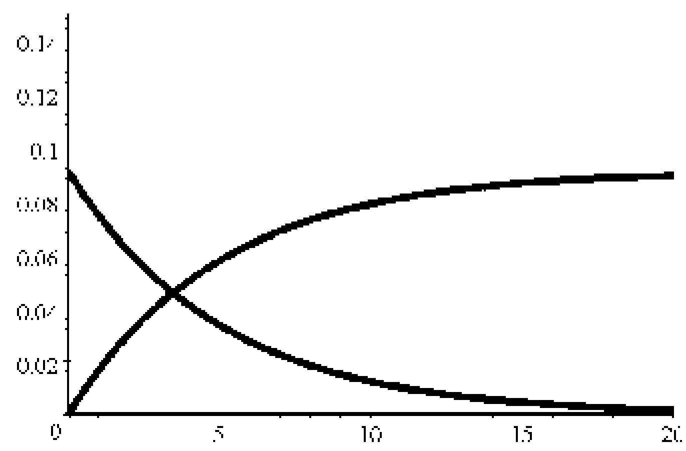
$$0 = \frac{U}{R} + A \Rightarrow A = -\frac{U}{R}$$

thus $\Rightarrow I = \frac{U}{R}\left(1 - e^{-\frac{R}{L}t}\right)$

for t=0 ', uncouple'':

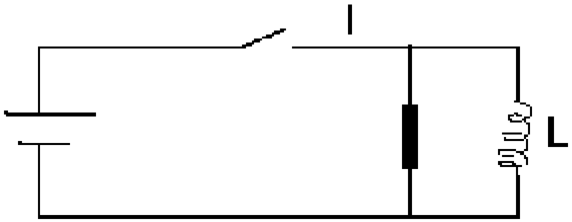
$t = 0, U = 0 \qquad \Rightarrow A = I_0$

and thus $I = I_0 \bullet e^{-\frac{R}{L}t}$



8.10. Energy and density of energy in a magnetic field

Energy of breaking current: From magnetic field of coil



W= energy

remember: Power P= $U \cdot I$

or with U=R · I

$$P = I^2 \cdot R$$

$$W = \int_0^\infty I^2 \cdot R \cdot dt = \int I_0^2 \cdot e^{-2\frac{R}{L}} \cdot R \cdot dt =$$

$$= R \cdot I_0^2 \cdot \int_0^\infty e^{-2\frac{R}{L}} \cdot dt = R \cdot I_0^2 \left[-\frac{L}{2R} e^{-\frac{2R}{L}t} \right]_0^\infty = \frac{1}{2} L \cdot I_0^2$$

Magnetic field energy of a coil

For an elongate coil: $L = \mu_0 \frac{N^2 \cdot A}{l}$

$$W = \frac{1}{2} \mu_0 \frac{N^2 \cdot A}{l} \cdot I^2 = \frac{1}{2} \underbrace{\mu_0 \frac{N \cdot I}{l}}_{B} \cdot \underbrace{\frac{N \cdot I}{l}}_{\frac{B}{\mu_0}} \cdot \underbrace{A \cdot l}_{\text{field volume}}$$

$$W = \frac{1}{2} \frac{B^2}{\mu_0} \cdot V \Rightarrow$$

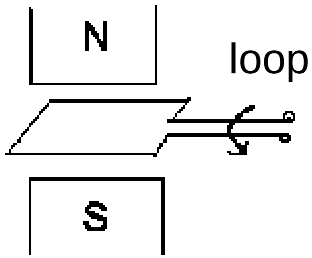
energy density: $w_M = \frac{B^2}{2 \cdot \mu_0}$

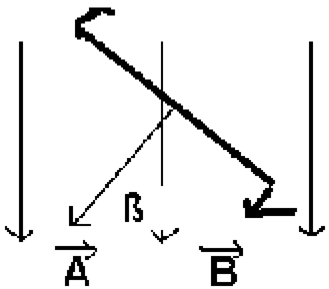
compared to : electric field

$$w_E = \frac{1}{2} \varepsilon_0 E^2$$

9. Alternating voltage and alternating current

9.1.Exp:





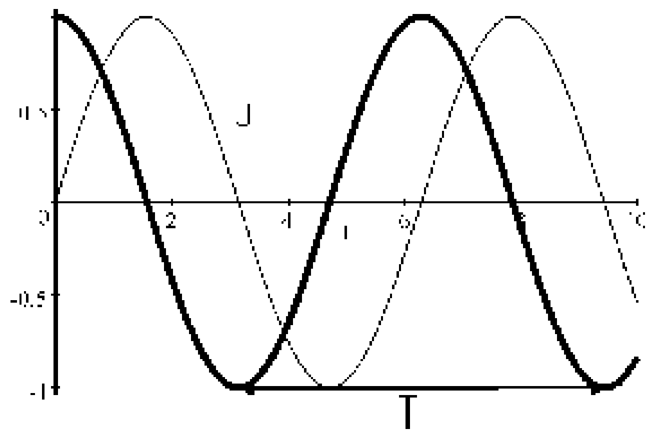
$\Phi = \vec{B} \cdot \vec{A}; A$: area of coil,B: field=const.

Turning the loop: $\Phi = B \cdot A \cdot \cos\beta;$ be $\beta = \omega t$

$$\Phi(t) = B \cdot A \cdot \cos\omega t \Rightarrow U_{ind} = U_{\sim} = U(t) = -\frac{d\Phi}{dt} = \omega \cdot B \cdot A \cdot \sin\omega t$$

$$U(t) = N \cdot \omega \cdot B \cdot A \cdot \sin\omega t$$

flux/voltage



$$\nu = \frac{1}{T}, \omega = 2\pi\nu$$

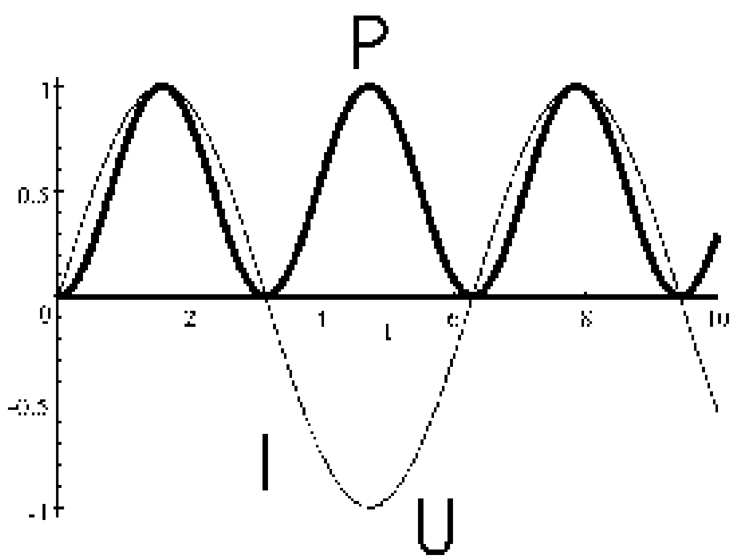
9.2. Effektiv values of current and voltage

$$\underbrace{I_{\sim}}_{\text{Momentary value}} = \frac{U_{\sim}}{R} = \underbrace{\frac{U_0}{R}}_{= I_0 \text{ (peak value)}} \sin \omega t$$

Momentary value = I_0(peak value)

Ohmic resistance: voltage and current in phase

Power consumption: $P = U \cdot I = U_0 \cdot \sin \omega t \cdot I_0 \cdot \sin \omega t = \frac{U_0^2}{R} \sin^2 \omega t$



Average: $\overline{P} = \frac{U_0^2}{R} \cdot \underbrace{\frac{1}{T} \int_0^T \sin^2 \omega t \cdot dt}_{0.5}$

$$= \frac{1}{2} \frac{U_0^2}{R} = \frac{1}{2} P_0$$

Def.: $\overline{P} = \frac{U_{eff}^2}{R}$ with

$$U_{eff} = \frac{1}{\sqrt{2}} U_0; I_{eff} = \frac{1}{\sqrt{2}} I_0$$

From socket: $U_{eff} = 220V \Rightarrow U_0 = 311V$

9.3. Alternate current resistors

Exp: Phase shift between current and voltage

Inductivity

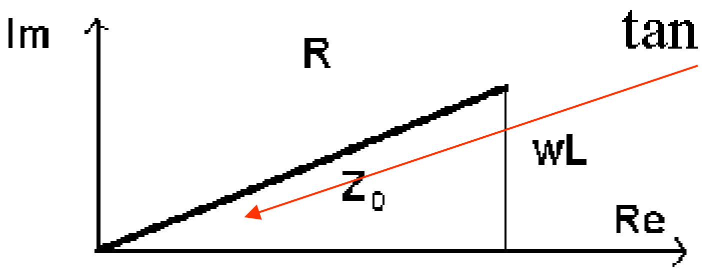
$$U = U_L + U_R = L \cdot \dot{I} + R \cdot I$$

Ansatz: $U = U_0 \cdot e^{i\omega t} \quad I = I_0 \cdot e^{i(\omega t - \varphi)} \quad \dot{I} = i\omega \cdot I_0 \cdot e^{i(\omega t - \varphi)} = i\omega \cdot I$

$$\Rightarrow U = \underbrace{(R + i\omega L)} \cdot I$$

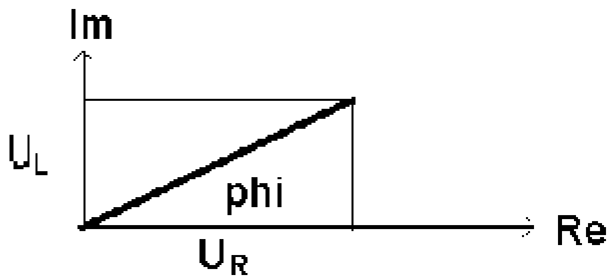
Complex resistor Z

$$U = Z \cdot I; Z = \frac{U}{I} = \frac{U_0}{I_0} e^{i\varphi} = Z_0 e^{i\varphi}$$



$$\tan\varphi = \frac{\omega L}{R}$$

For current and volage:



$$U_L = i\omega L \cdot I \Rightarrow$$

current lags behind current : φ

Power: $P(t) = U \cdot I = U_0 \cdot \cos\omega t \;\; I_0 \cdot \cos(\omega t - \varphi)$

Average: $\overline{P} = \frac{1}{T}\int_0^T P(t) \cdot dt$ $\quad \overline{P} = \frac{1}{2}P_0 \cdot \cos\varphi$

limiting cases:1) L=0 $\Rightarrow \varphi = 0$ 2) R=0 $\Rightarrow \varphi = 90^0$

current lags behind votage 90 ° nach $\quad \overline{P} = 0$ but idle power!

capacity:

$$U = U_c + U_R, Q = C \cdot U_C \Rightarrow U_C = \frac{1}{C} \cdot Q = \frac{1}{C}\int I \cdot dt$$

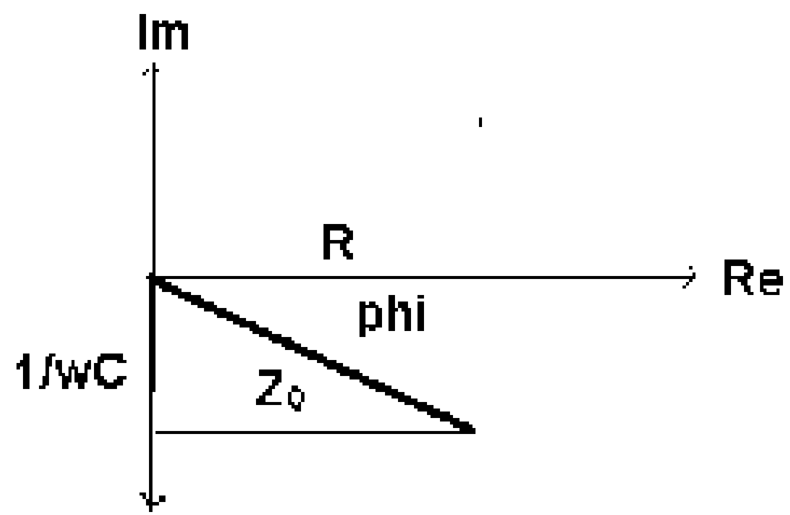
$$\Rightarrow U = \frac{1}{C}\int I \cdot dt + R \cdot I$$

$\dot{U} = \frac{1}{C}I + R \cdot \dot{I} \;\; *$ Ansatz: $U = U_0 e^{i\omega t} \Rightarrow \dot{U} = i\omega U_0 e^{i\omega t} = i\omega U$

$$I = I_0 e^{i(\omega t+\varphi)} \Rightarrow \dot{I} = i\omega I_0 e^{i(\omega t+\varphi)} = i\omega I \;\; * \Rightarrow i\omega U = \frac{1}{C}I + R \cdot i\omega I$$

$U = (R - i\frac{1}{\omega C})I; Z = (R - i\frac{1}{\omega C})$: complex resistor

$$Z = \frac{U}{I} = \frac{U_0}{I_0} e^{-i\varphi} = Z_0 e^{-i\varphi}$$



current is ahead of voltage!
again here

$$\overline{P} = \frac{1}{2} P_0 \cdot \cos\varphi$$

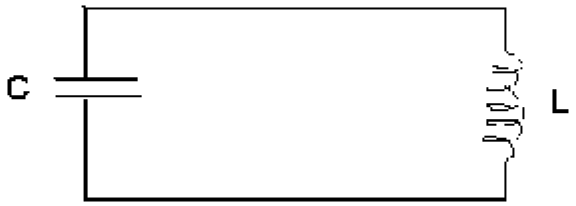
limiting case: R=0 current is ahead of voltage of 90 ° !

9.4. Electric oscillator circuit

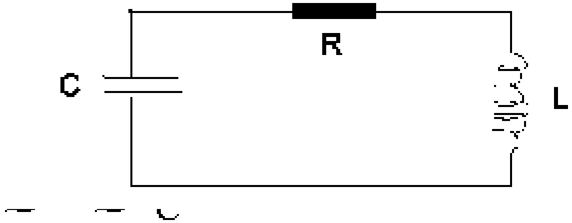
See also mechanics!

a) undamped oscillator circuit

Energiesatz:



$$\underbrace{W_C}_{\frac{1}{2}CU^2} + \underbrace{W_L}_{\frac{1}{2}LI^2} = \text{const.}$$



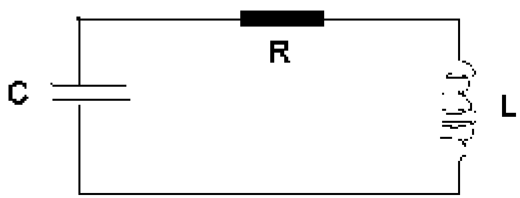
Differentiate after t:

$$\frac{1}{C}Q + L\dot{I} = 0$$

$$\frac{1}{C}\dot{Q} + L\ddot{I} = 0 \quad \Rightarrow \quad \boxed{\ddot{I} + \frac{1}{LC}I = 0}$$

Solution: $I = I_0 e^{i\omega_0 t}$ with $\omega_0 = \frac{1}{\sqrt{LC}}$

b) Damped oscillations



R: Energy pass to heat:

$$-\frac{d}{dt}\{W_C + W_L\} = I^2 R$$

$$-Q\dot{Q}\frac{1}{C} - L I\dot{I} = I^2 R$$

$$\frac{1}{C}Q + L\dot{I} + IR = 0$$

Differenciate: $\frac{1}{C}\dot{Q} + L\ddot{I} + R\dot{I} = 0$ or $L\ddot{I} + R\dot{I} + \frac{1}{C}I = 0$

Equation of oscillation for current!

Comparison with mechanics: $Q \doteq x; \vec{I} \doteq \vec{v};$

$E_E = \frac{1}{2}\frac{Q^2}{C}; E_B = \frac{1}{2}LI^2 \quad m\ddot{x} + k\dot{x} + Dx = 0 \Rightarrow L \doteq m, k \doteq R, \frac{1}{C} \doteq D$

Solution: $I = I_0 e^{-\delta t} e^{\pm i\omega t}; \delta = \frac{R}{2L}; \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$

$\omega = \omega_0 \sqrt{1 - \frac{CR^2}{4L}} \Rightarrow$ 3 cases: a) $R^2 \prec \frac{4L}{C}$: case of oscillation

b) $R^2 = \frac{4L}{C}$: Aperiodic critical damping $I = I_0 \cdot e^{-\delta t}$

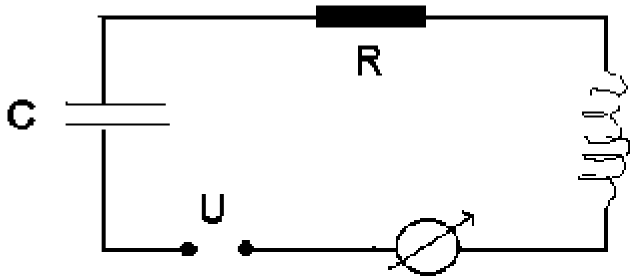
c) $R^2 \succ \frac{4L}{C}$: critical damping: ω gets imaginary $I = I_0 \cdot e^{-(\delta+\omega)t}$

quality factor: $Q = \frac{\omega}{2\delta}$ s. Mechanics! lecture 16

c) Forced oscillations

$U = U_0 \sin\omega t$

power balance:



$$\frac{d}{dt}\{W_C + W_L\} + I^2 R = U_0 \cdot I \cdot \sin\omega t$$

$$Q \cdot \dot{Q} \frac{1}{C} + L \cdot I \cdot \dot{I} + I^2 R = U_0 \cdot I \cdot \sin\omega t$$

$$Q \cdot \frac{1}{C} + L \cdot \dot{I} + I R = U_0 \cdot \sin\omega t \; :$$

differentiate

$$L \ddot{I} + R \dot{I} + \frac{1}{C} I = \omega \cdot U_0 \cdot \cos\omega t$$

$$\ddot{I} + \frac{R}{L} \cdot \dot{I} + \omega_0^2 \cdot I = \frac{1}{L} \omega \cdot U_0 \cdot \cos\omega t; *$$

solution wanted in the shape:

$$I = I_0 \sin(\omega t - \varphi) = I_0 \cdot [\sin\omega t \cos\varphi - \cos\omega t \sin\varphi]$$

$$\dot{I} = \omega \cdot I_0 \cos(\omega t - \varphi) = \omega \cdot I_0 [\cos\omega t \cos\varphi + \sin\omega t \sin\varphi]$$

$$\ddot{I} = -\omega^2 I_0 \sin(\omega t - \varphi) = -\omega^2 I_0 [\sin\omega t \cos\varphi - \cos\omega t \sin\varphi]$$

insert in $* : I_0[-\omega^2 \sin\omega t \cos\varphi + \omega^2 \cos\omega t \sin\varphi] +$

$$\frac{R}{L} I_0[\omega \cos\omega t \cos\varphi + \omega \sin\omega t \sin\varphi] +$$

$$\omega_0^2 I_0 \bullet [\sin\omega t \cos\varphi - \cos\omega t \sin\varphi] = \frac{1}{L}\omega \bullet U_0 \bullet \cos\omega t$$

true for "all" times → coefficients of
sin and cos on both sides of equation the same!

$$I_0[-\omega^2 \cos\varphi + \frac{R}{L}\omega \sin\varphi + \omega_0^2 \cos\varphi] = 0$$

$$I_0[\omega^2 \sin\varphi + \frac{R}{L}\omega \cos\varphi - \omega_0^2 \sin\varphi] = \frac{1}{L}\omega \bullet U_0$$

→ φ

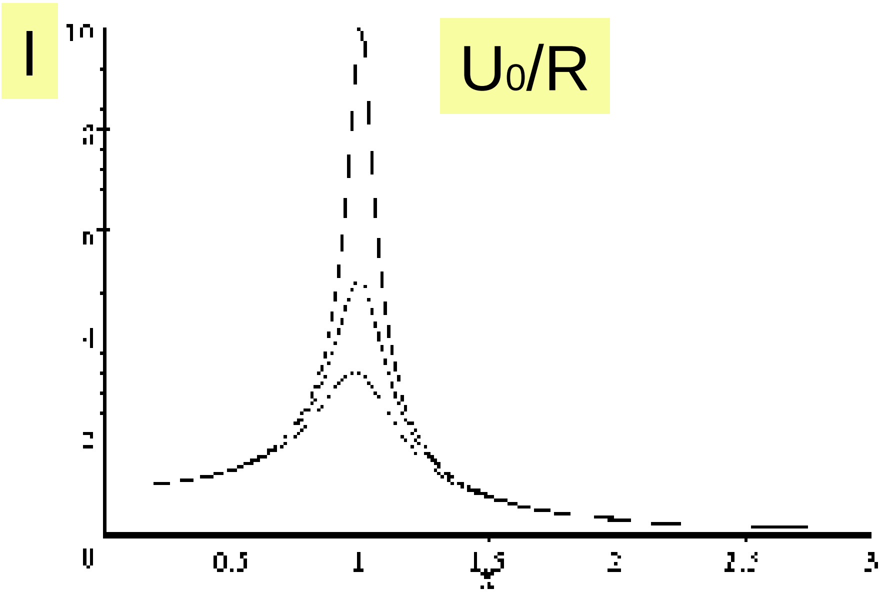
$$\tan\varphi = \frac{\omega^2-\omega_0^2}{\frac{R\omega}{L}}; \sin\varphi = \frac{\omega^2-\omega_0^2}{\sqrt{(\omega^2-\omega_0^2)^2+\frac{R^2\omega^2}{L^2}}}; \cos\varphi = \frac{\frac{R\omega}{L}}{\sqrt{(\omega^2-\omega_0^2)^2+\frac{R^2\omega^2}{L^2}}}$$

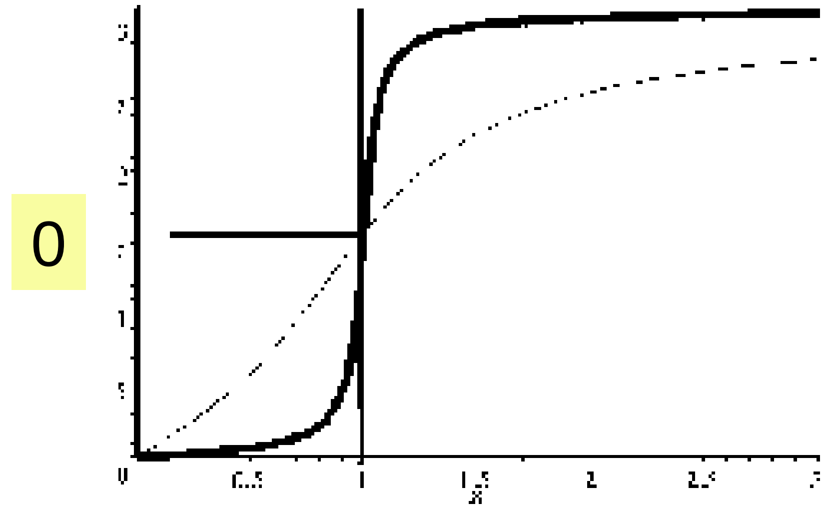
$$I_0 = \frac{\frac{1}{L}\omega \bullet U_0}{\sqrt{(\omega^2-\omega_0^2)^2+\frac{R^2\omega^2}{L^2}}}$$

at resonance:

$$\omega = \omega_0 = \frac{1}{\sqrt{L \bullet C}}, \varphi = 0$$

$$\Rightarrow I = \frac{U_0}{R} \bullet \cos\omega_0 t$$





voltage at R: $U_R = R \cdot I = U_0 \cdot \cos \omega_0 t$

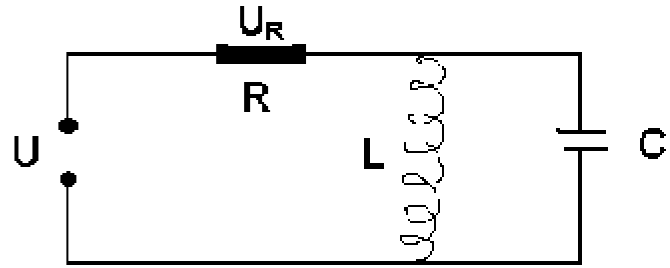
........at L: $U_L = L \cdot \dot{I} = -\omega_0 \cdot L \frac{U_0}{R} \sin \omega_0 t = \boxed{-\frac{U_0}{R}\sqrt{\frac{L}{C}} \sin \omega_0 t}$

........at C: $U_C = \frac{1}{C} Q = \frac{1}{C} \int I \cdot dt = \frac{U_0}{R \cdot C} \int \cos \omega_0 t \cdot dt$

$$= \frac{1}{\omega_0 \cdot C} \frac{U_0}{R} \sin \omega_0 t = \boxed{\frac{U_0}{R}\sqrt{\frac{L}{C}} \sin \omega_0 t}$$

in case of resonance $\boxed{U_L = -U_C}$

Parallelschwingkreis:



Gesamter Widerstand:

$$Z = R + Z_1 = Z_0 e^{i\varphi} \quad \frac{1}{Z_1} = \frac{1}{R_L} + \frac{1}{R_C} = \frac{1}{i\omega L} + i\omega C = i(\omega C - \frac{1}{\omega L})$$

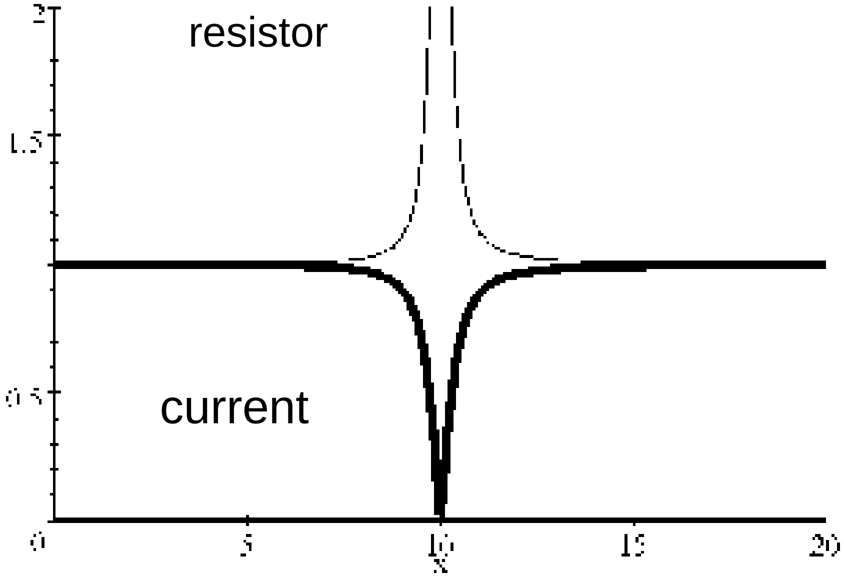
$$\Rightarrow Z_0 = \sqrt{R^2 + \frac{1}{(\frac{1}{\omega L} - \omega C)^2}} = R\sqrt{1 + \frac{\omega^2}{R^2(\frac{1-\omega^2 CL}{L})^2}} =$$

$$R\sqrt{1 + \frac{\omega^2}{R^2C^2(\frac{1-\omega^2 CL}{CL})^2}} = R\sqrt{1 + \frac{\omega^2}{R^2C^2(\omega_0^2-\omega^2)^2}}$$

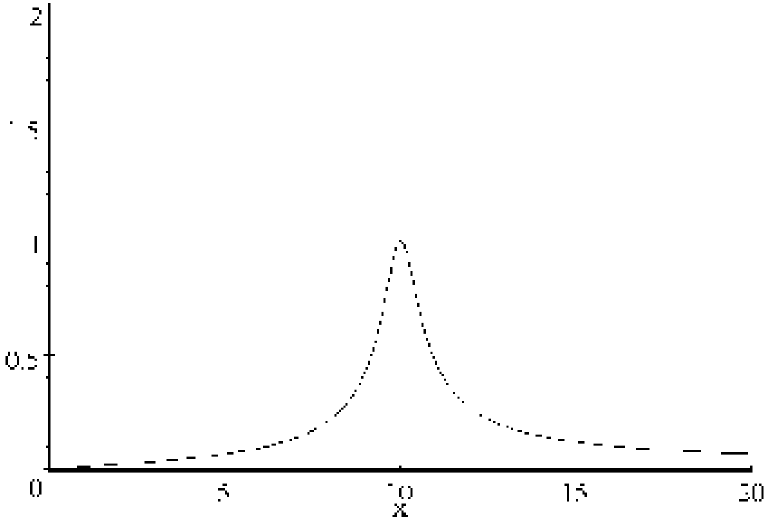
$$\varphi = \arctan\left[\frac{\frac{1}{\frac{1}{\omega L}-\omega C}}{R}\right] = \varphi = \arctan\left[\frac{\frac{\omega L}{1-\omega^2 CL}}{R}\right] =$$

$$\arctan\left[\frac{\frac{\omega L}{1-\frac{\omega^2}{\omega_0^2}}}{R}\right] = \arctan\left[\frac{\omega L\omega_0^2}{R(\omega_0^2-\omega^2)}\right] = \arctan\left[\frac{\omega}{RC(\omega_0^2-\omega^2)}\right]$$

Resonance: $\omega = \omega_0 \Rightarrow Z_0 \to \infty, I \to 0$



voltage:



i.e.: Filter in TV, Radio etc.