10. Electromagnetic waves

chapter

10.1. Repetition

Oscillation:(4.1.): $\ddot{\xi} + \omega_0^2 \cdot \xi = 0$

Periodic change in time

result: $\xi = \xi_0 \cdot e^{i\omega t}$

$\xi = \xi_0 \cdot \sin \omega t$

$\xi = \xi_0 \cdot \cos \omega t$

Waves: (4.5): Periodic change in time and space

$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2}$$

c: velocity

solution: $\xi = \xi(x,t) = f(x \pm ct), c = \frac{\Delta x}{\Delta t}$

specific:

$\xi = \xi_0 \cdot \sin[kx \pm \omega t]$

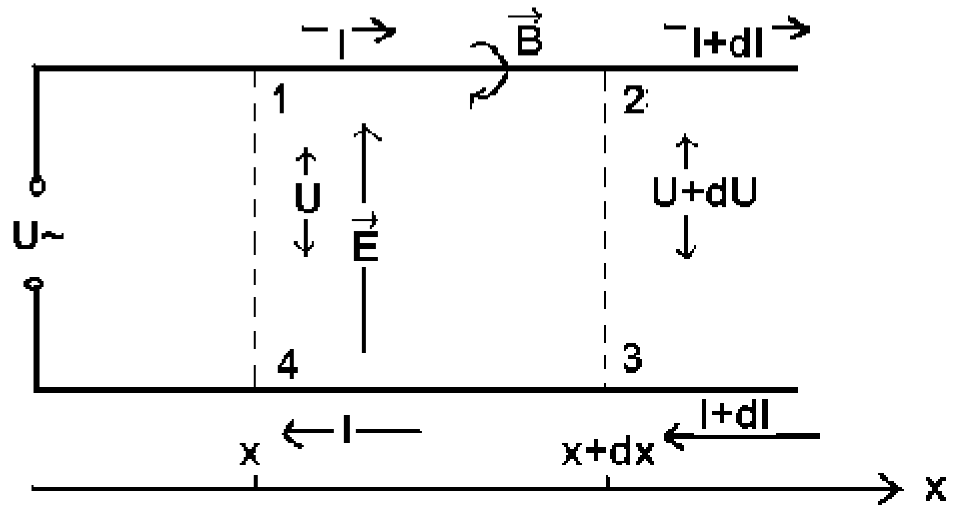
with "- " as outbound wave

with "+" as incoming wave

with wave number $k = \frac{2\pi}{\lambda}$ and angular frequency

$\omega = 2\pi\nu = \frac{2\pi}{T}$: $c = \lambda \cdot \nu$ and λ as wave length

10.2. Wave equation



charge balance on dx: (1-2)

influx: $dQ_1 = I(x) \cdot dt$

drain: $dQ_2 = I(x+dx) \cdot dt$

$= I(x) \cdot dt + \frac{\partial I}{\partial x} dx \cdot dt$

change of Q: $d^2Q = dQ_1 - dQ_2 = -\frac{\partial I}{\partial x} dx \cdot dt$

double ciruit line: Capacity $C = C^* \cdot dx;$ C^* stands for capacity/length

change of voltage: $[C^*] = \frac{F}{m}$

$$dU = \frac{d^2Q}{C^* \cdot dx} = -\frac{1}{C^*} \frac{\partial I}{\partial x} \cdot dt \Rightarrow \boxed{\frac{\partial U}{\partial t} = -\frac{1}{C^*} \frac{\partial I}{\partial x}}$$

a change of I leads to a change of $\vec{B}, \Phi$

$dU_{ind} = -L \frac{\partial I}{\partial t} = -L^* \cdot dx \cdot \frac{\partial I}{\partial t}$ with L^* as inductivity/length $[L^*] = \frac{H}{m}$

with $dU_{ind} = dU$ and for $R = 0$ $\boxed{\frac{\partial U}{\partial x} = -L^* \frac{\partial I}{\partial t}}$

$$\frac{\partial U}{\partial t} = -\frac{1}{C^*}\frac{\partial I}{\partial x}$$ $$\frac{\partial U}{\partial x} = -L^*\frac{\partial I}{\partial t}$$

Partial differentiation:

$$\frac{\partial^2 U}{\partial x \cdot \partial t} = -\frac{1}{C^*}\frac{\partial^2 I}{\partial x^2}$$ $$\frac{\partial^2 U}{\partial x \cdot \partial t} = -L^*\frac{\partial^2 I}{\partial t^2}$$

wave equation:

$$\frac{\partial^2 I}{\partial t^2} = \frac{1}{L^* \cdot C^*}\frac{\partial^2 I}{\partial x^2}$$

alike for voltage:

$$\frac{\partial^2 U}{\partial t^2} = \frac{1}{L^* \cdot C^*}\frac{\partial^2 U}{\partial x^2}$$

solution: $U(x,t) = U_0 \cdot \sin(kx - \omega t)$

as outgoing wave

with c= $\frac{1}{\sqrt{L^* \cdot C^*}}$

as phase velocity

for parallel conductor:

$$L^* \cdot C^* = \varepsilon_0 \cdot \mu_0$$

because $C^* = \frac{\varepsilon_0}{4\ln(\frac{d}{r})}$ and $L^* = 4\mu_0 \ln\frac{d}{r}$

count $c = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}}$, i.e. wave propagates with c along the line

Is there between the lines an ε or $\mu \Rightarrow$ $c = c_0 \frac{1}{\sqrt{\varepsilon \cdot \mu}}$

voltage distribution: $U(x,t) = U_0 \sin(kx - \omega t)$

current distribution: $I(x,t) = I_0 \sin(kx - \omega t + \varphi)$

because there could be a phase difference

$$\frac{\partial U}{\partial x} = k \cdot U_0 \cdot \cos(kx - \omega t)$$

from $\frac{\partial U}{\partial x} = -L^* \frac{\partial I}{\partial t}$

$$-L^* \frac{\partial I}{\partial t} = -L^* \cdot (-)\omega \cdot I_0 \cos(kx - \omega t + \varphi) \Longrightarrow$$

current and voltage are in phase!

$$k \cdot U_0 \cdot \cos(kx - \omega t) = L^* \cdot \omega \cdot I_0 \cos(kx - \omega t)$$

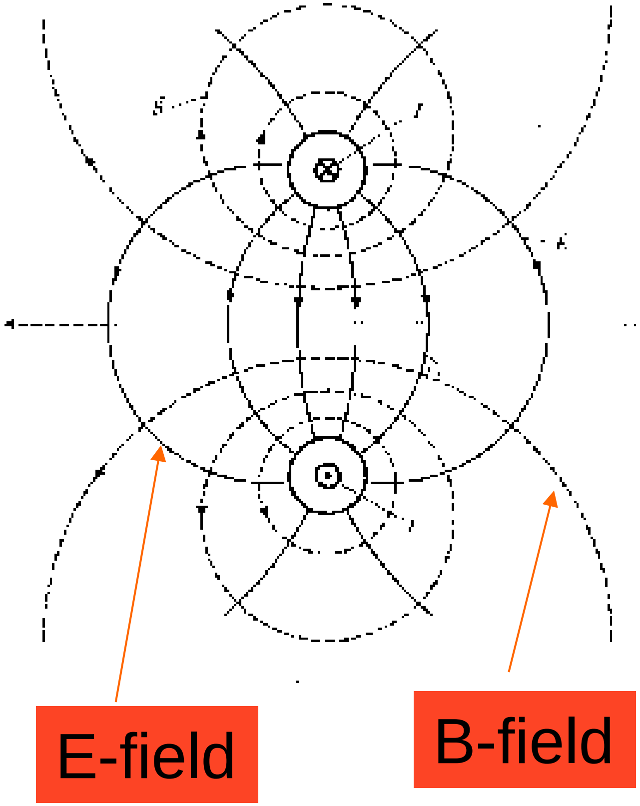
$$\frac{U_0}{I_0} = \frac{L^* \cdot \omega}{k} = L^* \cdot c = \frac{L^*}{\sqrt{L^* \cdot C^*}} = \sqrt{\frac{L^*}{C^*}}$$

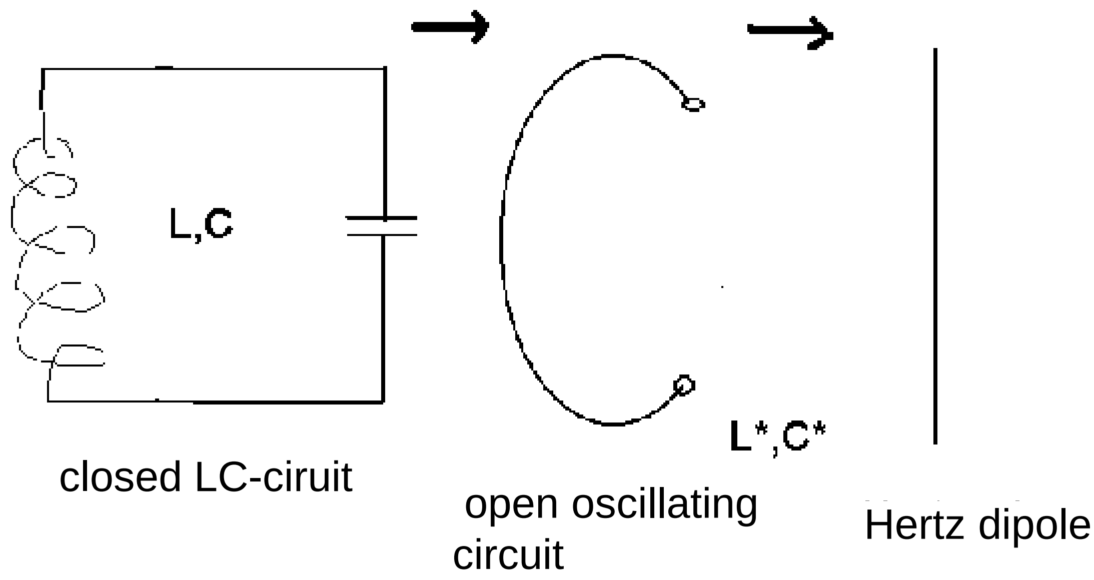
over the whole length of the line: $\sqrt{\frac{L^*}{C^*}}$: wave resistance

$\frac{U}{I} = \sqrt{\frac{L^*}{C^*}}$, by terminating the line on any place with R=$\sqrt{\frac{L^*}{C^*}}$ →

the energy transported in the wave gets absorbed from R without reflexion!

10.3 The radiation dipole and free waves



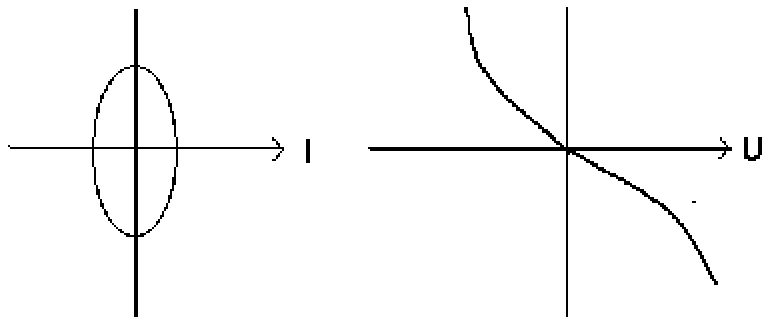


conditions for resonance $\nu = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$ or with Hertz dipole:

$\nu \approx \frac{1}{l\cdot\sqrt{L^*\cdot C^*}}$ l length of Hertz dipole

in a dielectric medium accordingly $\sim \frac{1}{l\sqrt{\varepsilon}}$

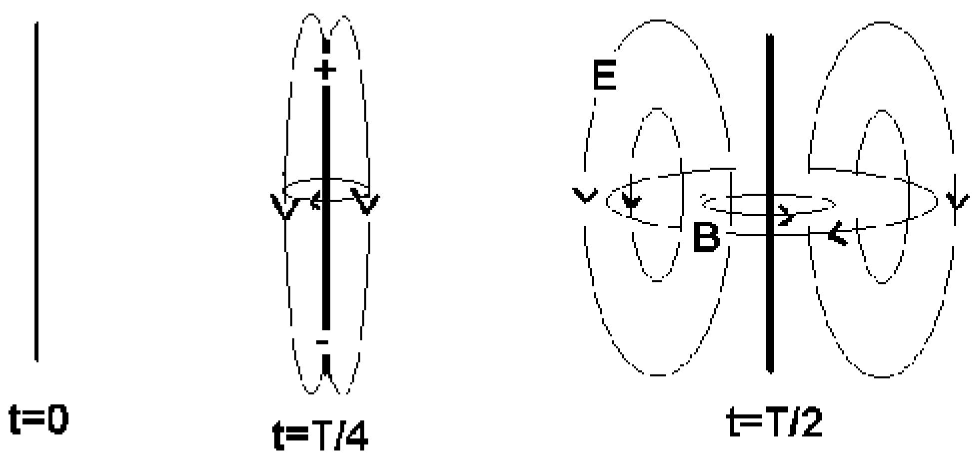
distribution of intensity of current and voltage:

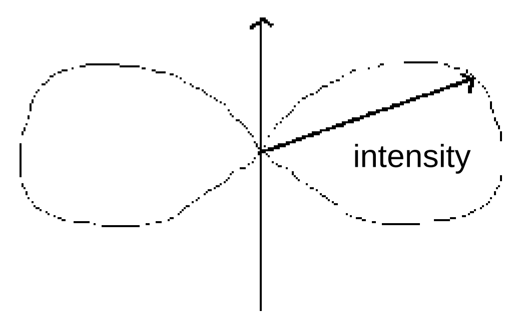


standing wave: wavelength is given by double length of the oscillator

$$\Rightarrow \nu = \frac{c}{\lambda} = \frac{c}{2l}$$ e.g: l=1.5m $$\Rightarrow \nu = \frac{3\cdot 10^8}{3} = 10^8 s^{-1}$$

characteristic of an el. dipole





axially symmetric around z-axis

maximal intensity ⊥ towards axis

10.4. Maxwell equations

so far:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$ Gauß $$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

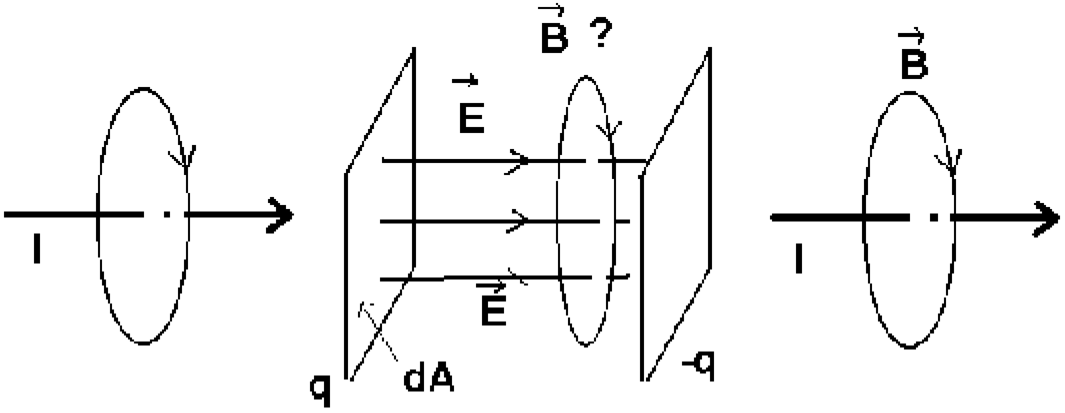
$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \cdot I$$ Ampère $$\vec{\nabla} \times \vec{B} = \mu_0 \cdot \vec{j}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$ Faraday $$\vec{\nabla} \times \vec{E} = -\dot{\vec{B}}$$

to understand electromagnetic waves
a 4. step is missing !

Ampère

Maxwells
displacements current:



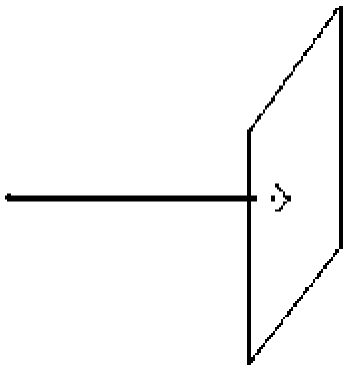
Maxwells displacement current:

$$\oint \vec{B} \cdot d\vec{s} = \underbrace{\mu_0 \cdot I_0}_{\text{Ampère}} + \underbrace{\mu_0 \cdot \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}}_{\text{Maxwell}}$$

Ampère

Maxwell

alternating field: q changes !



charged sheet

$$\Rightarrow \underbrace{\frac{dq}{dt}}_{\text{current}} = \underbrace{\varepsilon_0 \cdot \frac{d}{dt} \int \vec{E} \cdot d\vec{A}}_{I_{displacement}}$$

current $I_{displacement}$

in wire created

$\vec{B}$ $\Rightarrow$a time variable electric field $\frac{d\vec{E}}{dt}$

creates a magnetic eddy field

Maxwell: $\vec{\nabla} \cdot \vec{j} = 0$; static case

current conservation

$\vec{\nabla} \cdot \vec{j} + \frac{\partial \rho}{\partial t} = 0$: non static !

= using Gauss $(\vec{j} + \varepsilon_0 \cdot \dot{\vec{E}})$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

Gauß

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \cdot I + \mu_0 \cdot \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

Ampère +Maxwell

$$\vec{\nabla} \times \vec{B} = \mu_0 \cdot \vec{j} + \mu_0 \cdot \varepsilon_0 \cdot \dot{\vec{E}}$$

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A}$$

Faraday

$$\vec{\nabla} \times \vec{E} = -\dot{\vec{B}}$$

Synthesis of magnetism and electricity

Coulomb 1785 Force between charges

$\vec{E} - field$

Biot-Savart 1815, Ampère 1820-1825

Kraft zwischen zwei elektrischen Strömen

$\vec{B} - field$

Faraday 1831: a time dependent magnetic field
Induces an electric field

Maxwell 1865 : Displacement current

Wave equation of $\vec{E}$ and $\vec{B}$

In a- charge and current free vacuum:

$$\rho = 0 \qquad \vec{j} = 0$$

$$\vec{\nabla} \times \vec{E} = -\dot{\vec{B}} \qquad \vec{\nabla} \times \vec{B} = \mu_0 \cdot \varepsilon_0 \cdot \dot{\vec{E}}$$

$$\vec{\nabla} \times: \vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\vec{\nabla} \times \dot{\vec{B}} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla}\times: \vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\vec{\nabla}\times\dot{\vec{B}} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B}) = -\varepsilon_0\cdot\mu_0\frac{\partial^2\vec{E}}{\partial t^2}$$

with the vector relations: $\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \Delta\vec{E}; \Delta : Laplace$

$$\Delta\vec{E} = \varepsilon_0\cdot\mu_0\frac{\partial^2\vec{E}}{\partial t^2}; c = \frac{1}{\sqrt{\varepsilon_0\cdot\mu_0}}$$

for the x-component $\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \frac{1}{c^2}\frac{\partial^2 E_x}{\partial t^2}$

ε_0 : From measurements of Coulomb force between charges

μ_0 : From measurements of the force beween current-carrying power wires

Examples: Electromagnetic waves

$\nu = e.g.: 50\ Hz$ $\lambda \succ 10^6 m$: technical alternating current

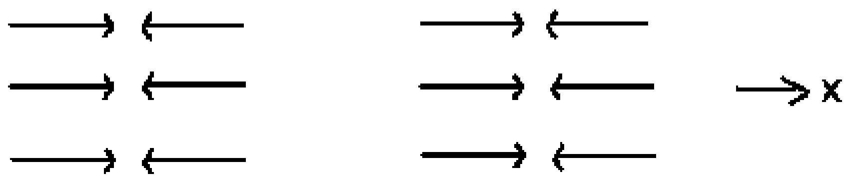
100MHz UKW

$\nu = 10^{22} Hz$ $\lambda = 10^{-14} m$: $\gamma -$ Radiation

Distances on a atomic or smaller scale: Quantummechanics+ spe
relativity as synthesis: Quantum fieldtheory

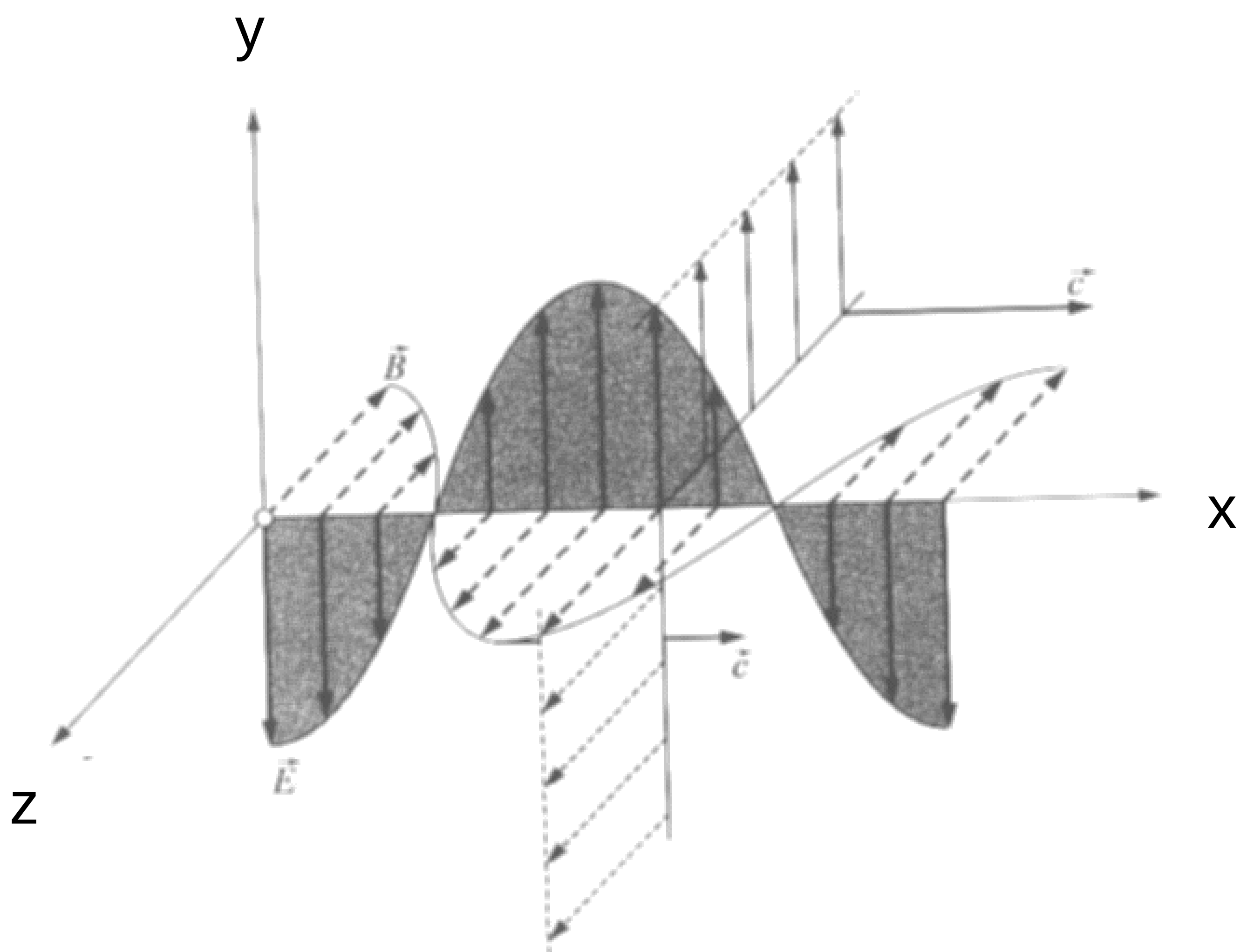
Comonents of the fields

From: $\int_A \vec{E} \cdot d\vec{A} = 0, \qquad \int_A \vec{B} \cdot d\vec{A} = 0 \Longrightarrow$



The electric (magnetic) field vector has no longitudinal - component

Propagation of transversal electromagnetic waves, only.



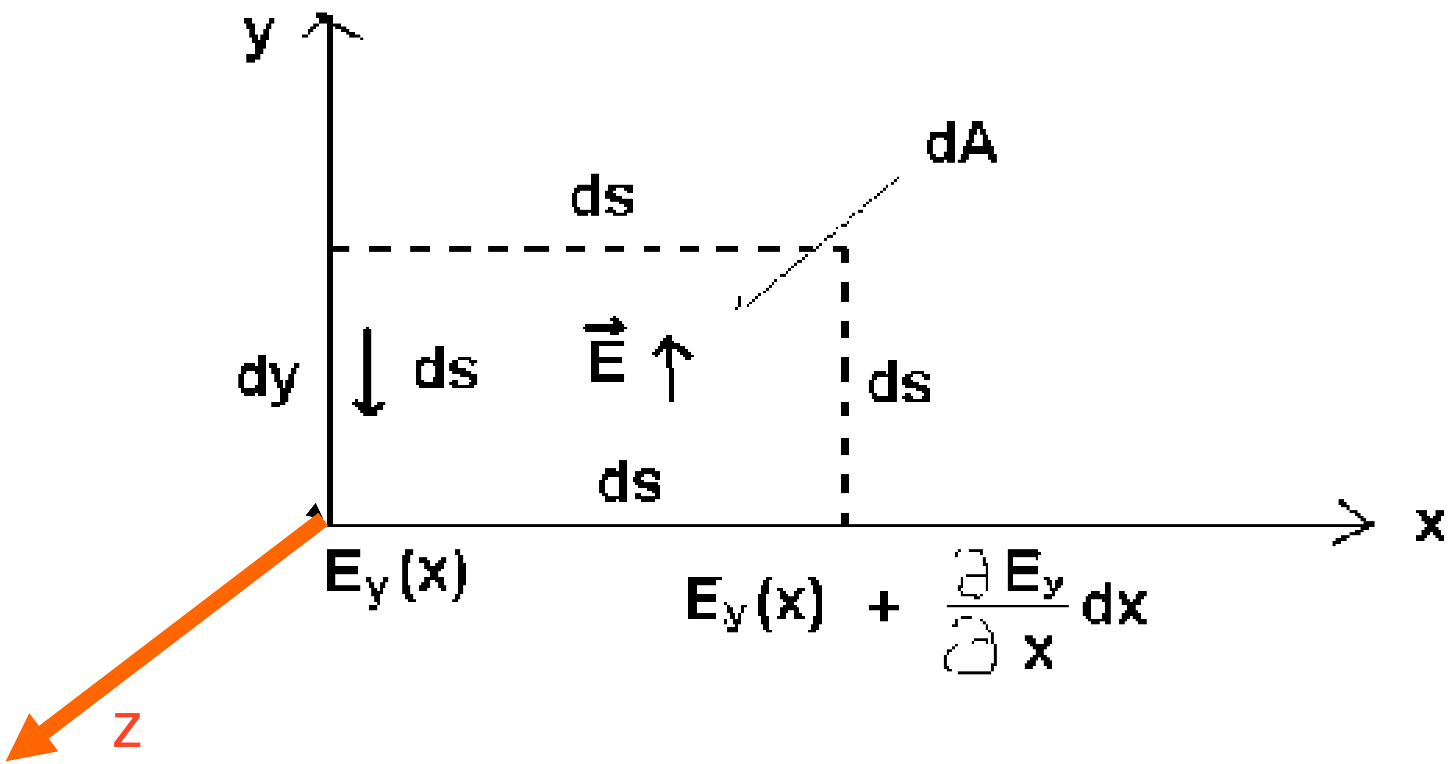
y
x
z
$\vec{B}$
$\vec{E}$
$\vec{c}$
$\vec{c}$

$$\oint \vec{E} \bullet d\vec{s} = -\frac{d}{dt}\int \vec{B} \bullet d\vec{A}$$

Faraday

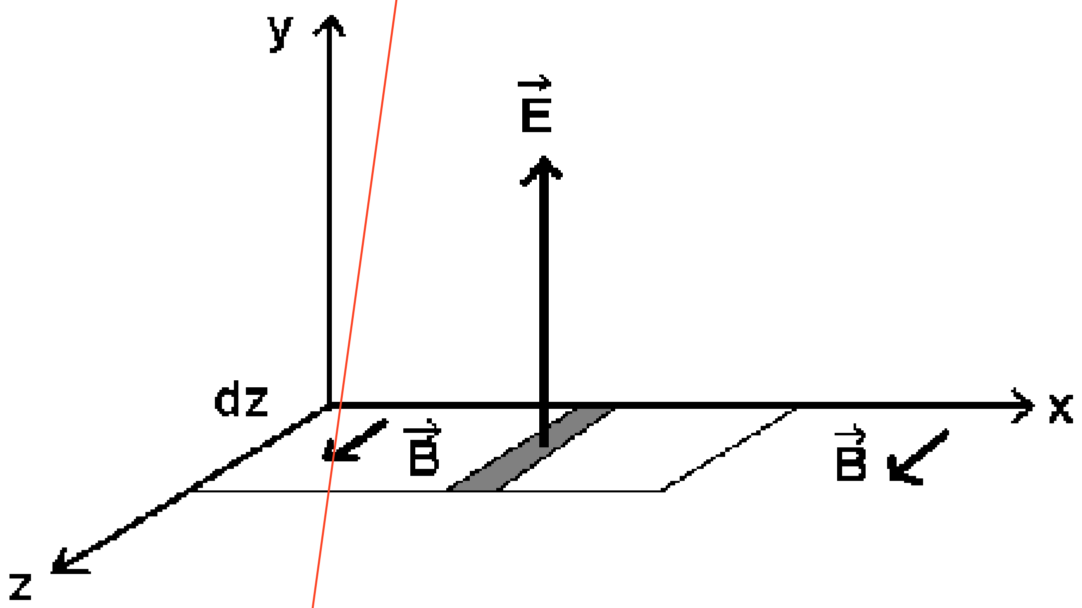
$$= E_y(x+dx) \bullet dy - E_y(x) \bullet dy \quad = (E_y(x) + \frac{\partial E_y}{\partial x}dx)dy - E_y(x) \bullet dy$$

$$= \frac{\partial E_y}{\partial x}dy \bullet dx = -\frac{\partial B_z}{\partial t}dx \bullet dy \Longrightarrow \qquad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \cdot I + \mu_0 \cdot \varepsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}$$

Ampère+Maxwell



$$\oint \vec{B} \cdot d\vec{s} = B_z \cdot dz - (B_z + \frac{\partial B_z}{\partial x} dx) \cdot dz$$

$$= -\frac{\partial B_z}{\partial x} dx \cdot dz = \mu_0 \cdot \varepsilon_0 \cdot dx \cdot dz \frac{\partial E_y}{\partial t}$$

or

$$\frac{\partial B_z}{\partial x} = -\varepsilon_0 \cdot \mu_0 \cdot \frac{\partial E_y}{\partial t}$$

$$\Longrightarrow \frac{\partial^2 B_z}{\partial x \partial t} = -\varepsilon_0 \cdot \mu_0 \cdot \frac{\partial^2 E_y}{\partial t^2}$$

and from $\boxed{\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}}$ $\Rightarrow \frac{\partial^2 E_y}{\partial x^2} = -\frac{\partial^2 B_z}{\partial x \partial t}$

ω

$\frac{\partial^2 E_y}{\partial x^2} = \varepsilon_0 \cdot \mu_0 \cdot \frac{\partial^2 E_y}{\partial t^2}$; solution: $E_y = E_{y_0} \cdot \sin(t - \frac{x}{c})$ $c = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}}$

likewise $\frac{\partial^2 B_z}{\partial x^2} = \varepsilon_0 \cdot \mu_0 \cdot \frac{\partial^2 B_z}{\partial t^2}$

Integration:

$\frac{\partial E_y}{\partial x} = -E_{y_0} \cdot \frac{\omega}{c} \cdot \cos \omega(t - \frac{x}{c}) = -\frac{\partial B_z}{\partial t}$ $B_z = \frac{E_{y_0}}{c} \sin \omega(t - \frac{x}{c})$

Electric and magnetic field have
at the same time and at the same place their maximum

$\vec{E}$ and $\vec{B}$ are in phase!

We observe: $\frac{E_y}{B_z} = c = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}}$

independent of the wavelength!

e.g.: Intensity of sunligth

E-field: 800V/m, B?

$B = \frac{E}{c} = \frac{800 V/m}{3 \cdot 10^8 m/s} = 2.67 \cdot 10^{-6} T$

Comparison: Static field of the earth:0.5 $\cdot 10^{-4} T$