10.6. Energy density of an electromagnetic wave and the Poynting - Vector

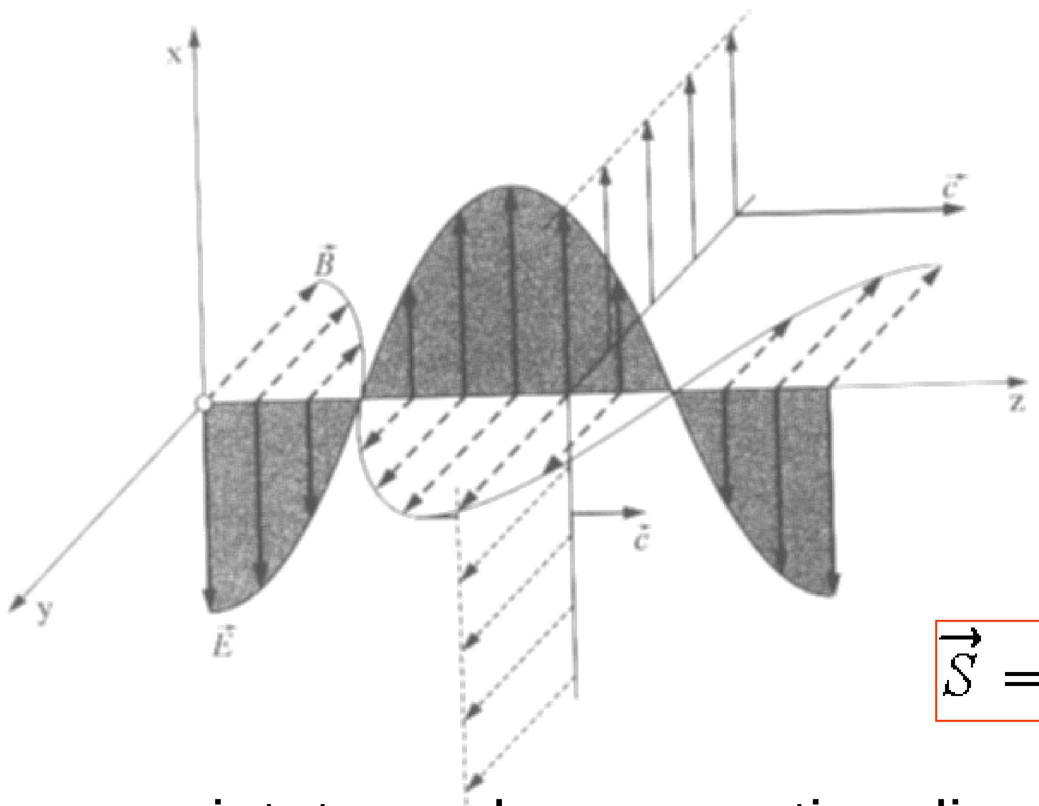
Energy density w: $w = \frac{\varepsilon_0}{2} E_y^2 + \frac{1}{2\mu_0} B_z^2$ $B_z^2 = \varepsilon_0 \cdot \mu_0 \cdot E_y^2$

$$\boxed{w = \varepsilon_0 \cdot E_y^2}$$

Energy current density:

$S = w \cdot c,$

pro unity of time and unity of area perpendicular crossing energy (intensity)



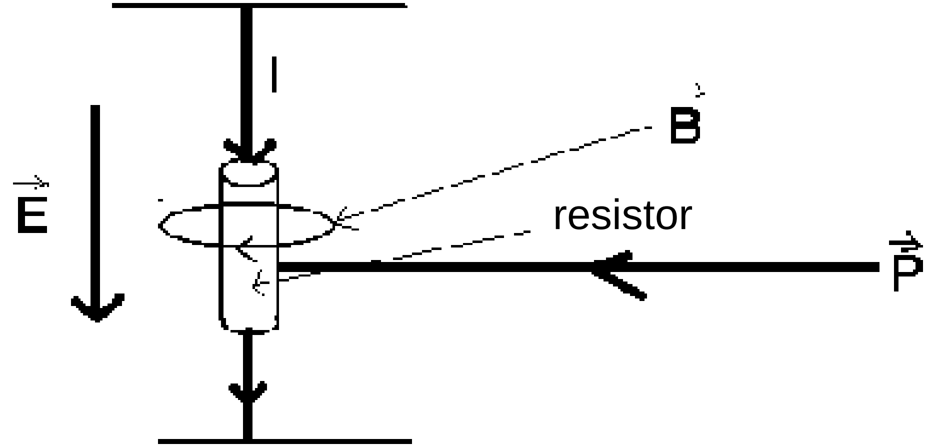
$$|\vec{E} \times \vec{B}| = \frac{E_y^2}{c}$$

$$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}}$$ Poyntingvektor

point towards propagation direction

e.g.: resistor

$\vec{S} \Rightarrow$



Energyflux from all sides
(Poynting vektor)
yields the quantity of heat $\boxed{I^2 \cdot R}$

10.7. Electromagnetic waves in a dielectric

refraction index

Vacuum + charges of plarisation
+ Ampère circle currents → Materie

Here only: $\vec{P} = \chi \cdot \varepsilon_0 \cdot \vec{E} = \varepsilon_0 \cdot (\varepsilon - 1) \cdot \vec{E}$ $\quad \varepsilon_0 \varepsilon \cdot \vec{E} = \varepsilon_0 \cdot \vec{E} + \vec{P}$

$$\int_A (\vec{E} + \frac{\vec{P}}{\varepsilon_0}) \cdot d\vec{A} = \int_A \varepsilon \cdot \vec{E} \cdot d\vec{A} = 0$$

In addition: $\int_A \vec{B} \bullet d\vec{A} = 0$ dazu : $\oint \vec{E} \bullet d\vec{s} = -\frac{d}{dt} \int_A \vec{B} \bullet d\vec{A}$

Remains unchanged

but $\oint \vec{B} \bullet d\vec{s} = \mu_0 \int_A \varepsilon_0 \frac{\partial}{\partial t} (\vec{E} + \frac{\vec{P}}{\varepsilon_0}) \bullet d\vec{A}$

$$= \mu_0 \bullet \varepsilon_0 \bullet \varepsilon \frac{d}{dt} \int \vec{E} \bullet d\vec{A}$$

Formal: ε_0 Gets replaced by $\boxed{\varepsilon_0 \bullet \varepsilon}$ also $\boxed{\mu_0 \rightarrow \mu \bullet \mu_0}$

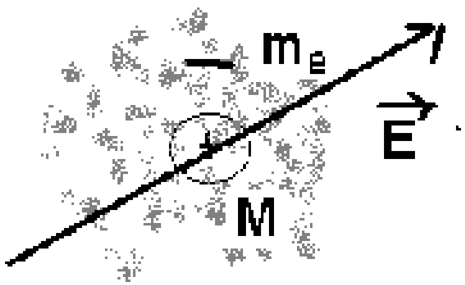
e.g: for $\frac{E}{B} = \frac{1}{\sqrt{\varepsilon \bullet \mu \bullet \varepsilon_0 \bullet \mu_0}} = \frac{c}{\sqrt{\varepsilon \bullet \mu}} = v; \varepsilon \gtrsim 1, \mu \succeq 1$

$\sqrt{\varepsilon} = n$ index of refraction $\mu = 1$

Optics

Be reminded!

Polarisation of atoms in electric alternating fields:



$\vec{E}$: outside field, masses M, m_e with M >>m_e

Force brings electron out of balance $F = q \bullet E$

$$m_e \frac{d^2x}{dt^2} + m_e \bullet \omega_0^2 \bullet x = q \bullet E_x^0 \bullet \cos \omega t$$

Solution with $x=x_0 * \cos\omega t$

$$\omega^2 \bullet m_e \bullet x_0 \bullet \cos \omega t + m_e \bullet \omega_0^2 \bullet x_0 \bullet \cos \omega t = q \bullet E_x^0 \bullet \cos \omega t$$

$$x_0 = \frac{q \cdot E_x^0}{m_2(\omega_0^2 - \omega^2)}$$

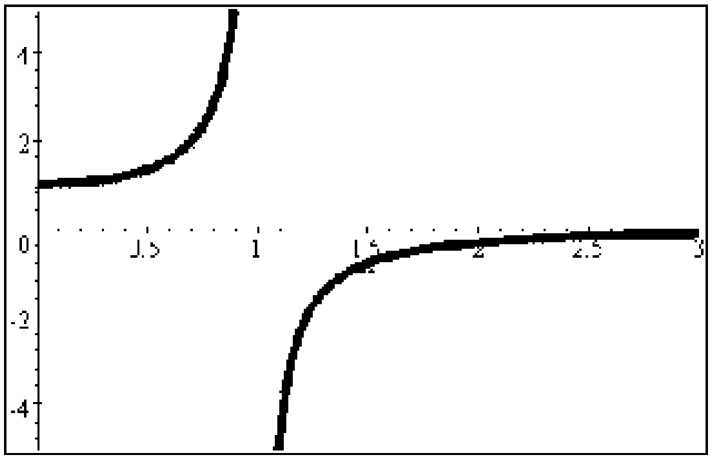
displacement corresponds to an oscillating dipolmoment

$$p_x = q \bullet x$$

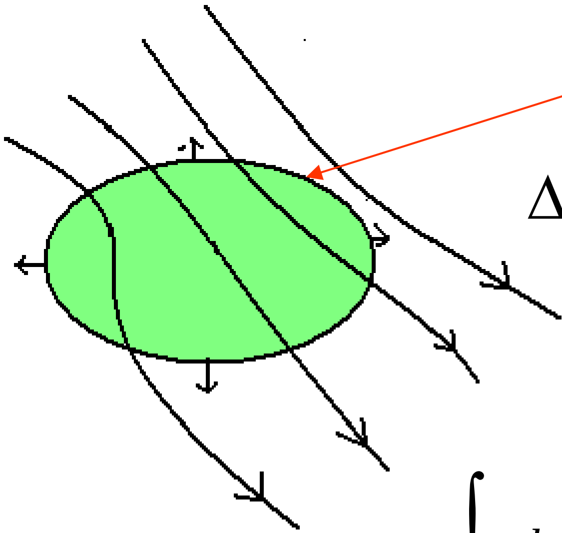
$m_e=m_2$

$$\Rightarrow p_x = \frac{q^2 \cdot E_x}{m_2(\omega_0^2 - \omega^2)} = \varepsilon_0 \bullet \alpha(\omega) \bullet E_x$$

ε



Modifications of Maxwell - equations in matter by electric polarisation



Integration includes charge!

$$\Delta Q_{pol} = -\int_{surface} \overrightarrow{P} \cdot \overrightarrow{dA} \overset{Gauß}{=} -\int_{volume} \nabla \cdot \overrightarrow{P} \cdot dV$$

$$\int_{volume} \rho_{pol} \cdot dV \rightarrow\rightarrow\rightarrow \rho_{pol} = -\nabla \cdot \overrightarrow{P}$$

$$\underbrace{\rho_0}_{\text{charges}} + \rho_{pol} = \rho \qquad \nabla\vec{E} = \frac{\rho_0+\rho}{\varepsilon_0} = \frac{\rho_0}{\varepsilon_0} - \frac{1}{\varepsilon_0}\nabla\vec{P}$$

$$\nabla\underbrace{\left(\varepsilon_0\vec{E} + \vec{P}\right)}_{\vec{D}} = \rho_0$$

$\vec{P}$ changes with time $\to\to\to$ charges change with time

$\to\to\to$ current

$$j = N \cdot q_e \cdot \underbrace{v}_{= \frac{dx}{dt}} \to j_{pol} = \frac{dP}{dt}$$

$$\to\to\to\to \; j = j_0 + j_{pol}$$

$$\boxed{c^2 \nabla \times \vec{B} = \frac{\vec{j_0}}{\varepsilon_0} + \frac{\vec{j_{pol}}}{\varepsilon_0} + \dot{\vec{E}}}$$

$$\varepsilon_0 \cdot c^2 \nabla \times \vec{B} = \vec{j_0} + \frac{\partial}{\partial t} \underbrace{\left(\varepsilon_0 \vec{E} + \vec{P} \right)}_{\vec{D}}$$

$$\boxed{\nabla \times \vec{B} = \mu_0 \cdot \vec{j_0} + \mu_0 \cdot \frac{\partial}{\partial t} \vec{D}}$$

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ and $\nabla \cdot \vec{B} = 0$ stand!

What happens with magnetic matter?

$$\vec{B} = \vec{B_0} + \underbrace{\mu_0 \cdot \vec{M}}_{\text{magnetisation}}$$

Does one want to have on the left side of Maxwell equation fields and on the right side currents, $\vec{H}$ gets introduced.

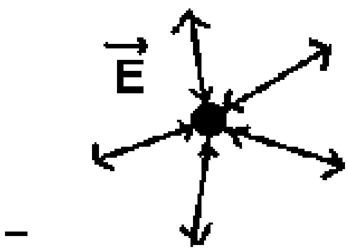
$$\vec{H} = \vec{B} - \mu_0 \cdot \vec{M}$$

$$\boxed{\nabla \times \vec{H} = \mu_0 \cdot \vec{j_0} + \mu_0 \cdot \frac{\partial}{\partial t}\vec{D}}$$

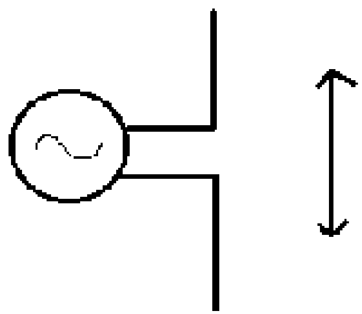
10.8. Polarisation

Polarisation of elektromagnetic wave is defined as the plane, which is spanned by the direction of the field $\vec{E}$ and the direction of propagation!

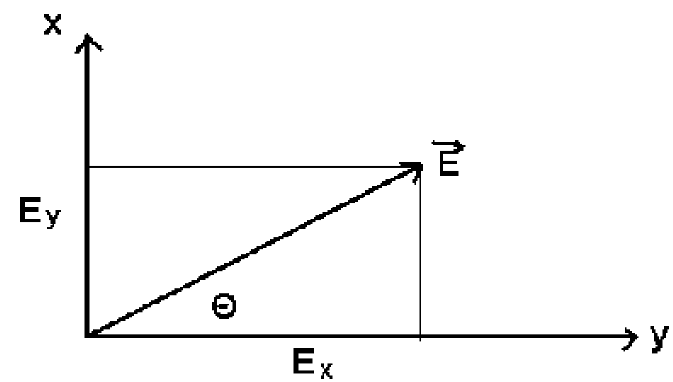
light bulb



high frequency



segmentation of the electric field in components



For intensities one has:

$$I_x \sim (E \cdot \cos \Theta)^2$$

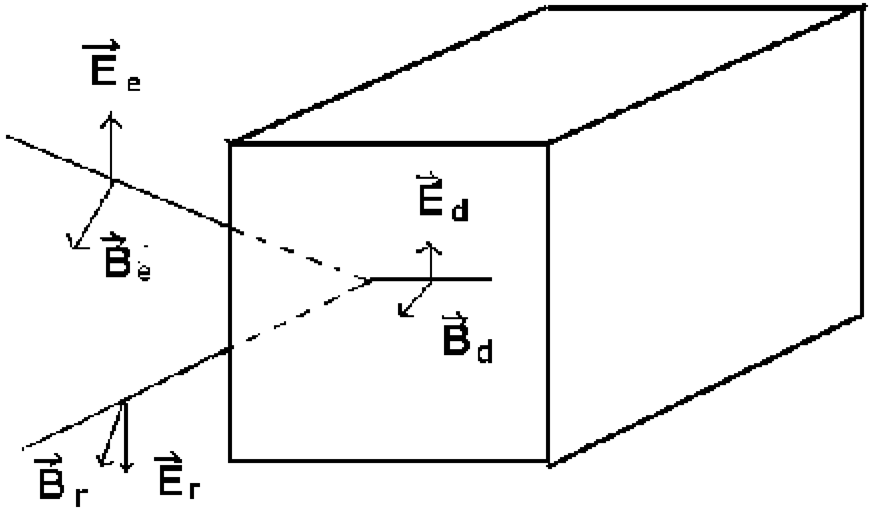
$$I_y \sim (E \cdot \sin \Theta)^2$$

10.9. Reflexion of a wave on a surface of an insulator

Example glass: almost perpendicular inclination
Reflexion, traversing beam

$\frac{E}{B} = c$ in vacuum and

$\frac{c}{n}$ in glass



Boundaries: equality of tangential field strength!

$$B_d = B_e + B_r \qquad E_d = E_e - E_r$$

Reflexion coefficient: $R = \frac{E_r}{E_e}$ mit $\frac{E}{B} = \frac{c}{n}$

$$E = \frac{c}{n}B, B = \frac{n}{c}E \implies \frac{E_e}{c} + \frac{E_r}{c} = \frac{n}{c}E_d = \frac{n}{c}(E_e - E_r); *$$

$$E_e - n \bullet E_e = -E_r - n \bullet E_r$$

$$\implies \boxed{R = \frac{E_r}{E_e} = \frac{n-1}{n+1}}$$

respectively: transmission factor D: $D = \frac{E_d}{E_e}$

$$\boxed{D = \frac{E_d}{E_e} = \frac{2}{n+1} = 1 - R}$$

reflectance $r = R^2$; transmission $d = D^2$

For intensities

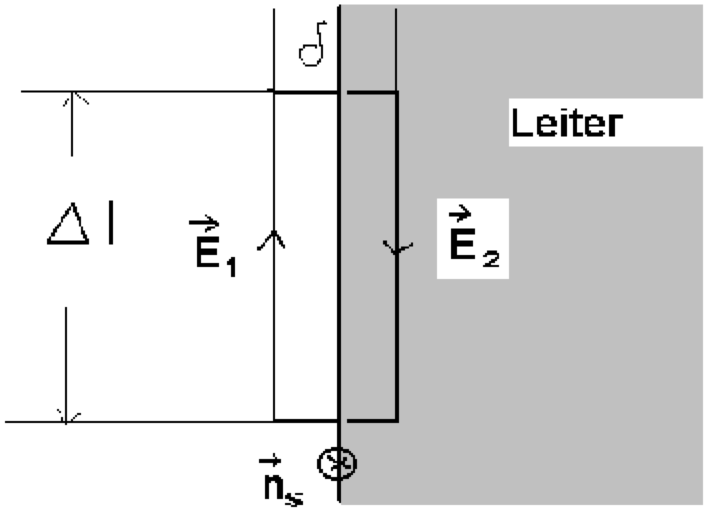
$$R^2 = (\frac{n-1}{n+1})^2 = \frac{n^2-2n+1}{n^2+2n+1} \qquad D^2 = \frac{4}{n^2+2n+1} \Rightarrow$$

glass with n=1.5 :$R^2 = \frac{.5^2}{2.5^2} = .04$ At a perpenticular incidence

In general: Fresnel‘s formulae of optics

10.10 Randbedingungen

Vakuum →perfekter Leiter



da $\vec{B} \cdot \vec{n}_s$ endlich!

Perfekter Leiter:

$$\sigma \to \infty \Rightarrow \vec{E} = 0$$

Zeitabhängigkeit von B:

$$B \sim e^{i\omega t}$$

$$\oint \vec{E} \cdot d\vec{s} = -i\omega \int \vec{B} \cdot \vec{n}_s \cdot \underbrace{dA}_{\Delta l \cdot \delta}$$

für $\delta \to 0 \Rightarrow \underbrace{(E_1 - E_2)}_{=0} \cdot \Delta l = 0$

(perfekter Leiter)

Die Tangentialkomponente des elektrischen Feldes verschwindet an der Oberfläche eines perfekten Leiters

Totale Reflexion der Welle

How does it look with $\vec{B}$? $\vec{B} = 0$ in conductor , because of $\vec{E}_2 = 0$

$$\oint \vec{E} \cdot d\vec{s} = -i\omega \int \vec{B} \cdot \vec{n}_s \cdot dA$$

with

$$\oint \vec{B} \cdot d\vec{s} = \int \sigma \cdot \vec{E} \cdot \vec{n}_s \cdot dA \qquad (B_1 - B_2) \cdot \Delta l = \sigma \cdot E \cdot \underbrace{dA}_{\Delta l \cdot \delta}$$

on the boundary →
surface current because $B_1 \neq 0$

Group velocity:

With that velocity, energy will be transported!

Special solution of the wave equation:

$$u(x,t) = Ae^{ikx-i\omega t} + Be^{-ikx-i\omega t}, k = \frac{2\pi}{\lambda}, \omega = \frac{2\pi}{T}$$

Linear superposition!

$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) e^{ikx-i\omega(k)t} dk$$

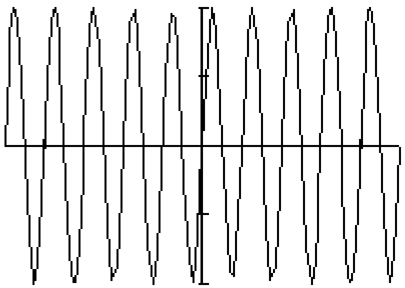
special for t=0

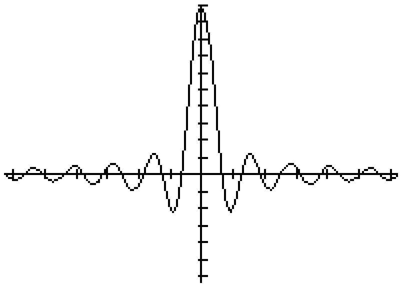
$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x,0) e^{-ikx} dx$$

$$u(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) \bullet e^{ikx} \bullet dk$$

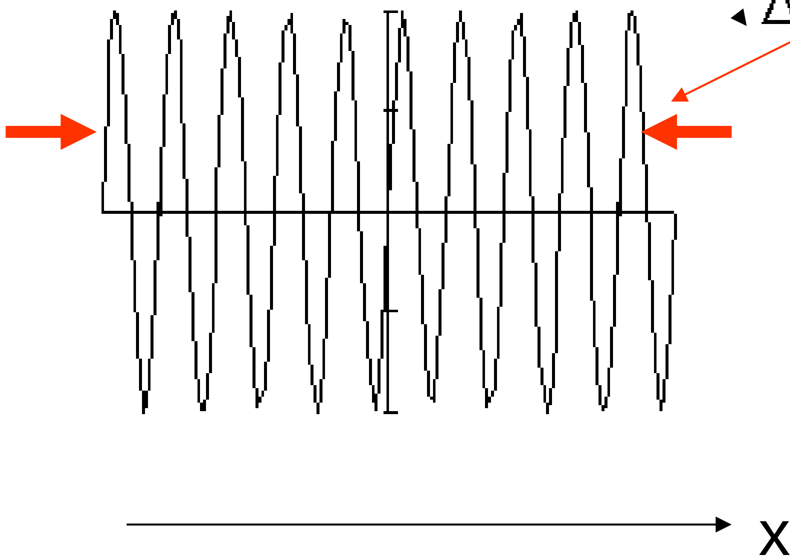
it is necessary $\Delta x \bullet \Delta k \geq \frac{1}{2}$

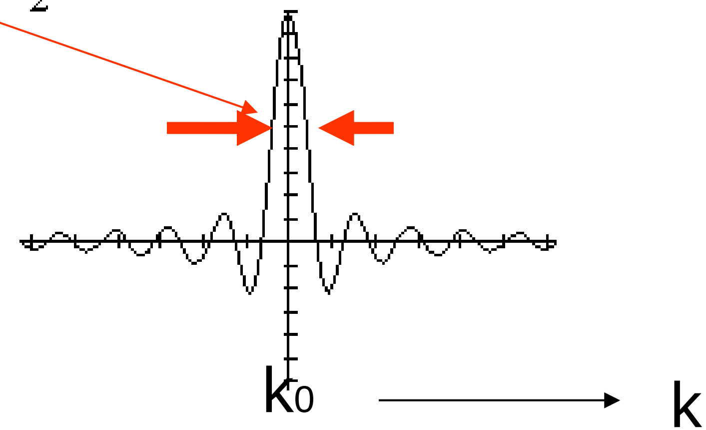
final wave train:





$\Delta x \cdot \Delta k \geq \frac{1}{2}$





$\rightarrow\rightarrow\rightarrow \; v_{group} = (d\omega/dk)\,|_0$

$\omega(k)!$ If A(k) close around k_0 concentrated

Compared phase:

$0 \leq v_{Phase} \leq \infty$

$$\omega(k) = \omega_0 + \frac{d\omega}{dk}\,|_0\,(k-k_0) + \ldots..$$

$$u(k) \approx \frac{e^{i[k_0(d\omega/dk)|_0 - \omega_0]\cdot t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) e^{i[x-(d\omega/dk)|_0 \cdot t]k}\,dk$$

true only u(x',0)
with

$$x' = x - (d\omega/dk)\,|_0 \cdot t$$

$\rightarrow\rightarrow\rightarrow \; v_{group} = \left(\frac{d\omega}{dk}\right)|_0$

group velocity

$v_{Gruppe} \precsim c$ is always true!