## Group velocity

Energy gets transported with this velocity!

Special solution of wave equation:

$$u(x,t) = Ae^{ikx-i\omega t} + Be^{-ikx-i\omega t}, k = \frac{2\pi}{\lambda}, \omega = \frac{2\pi}{T}$$

Linear superposition!

$$u(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) e^{ikx-i\omega(k)t} dk$$

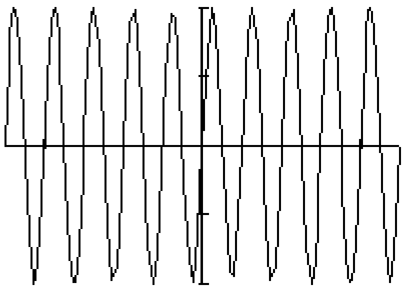
special for t=0

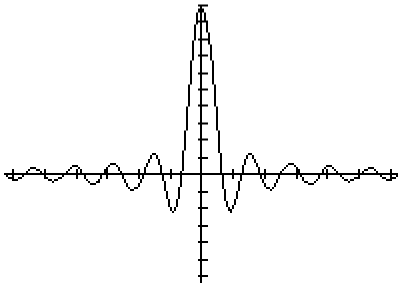
$$u(x,0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) \cdot e^{ikx} \cdot dk$$

we have $\Delta x \cdot \Delta k \geq \frac{1}{2}$

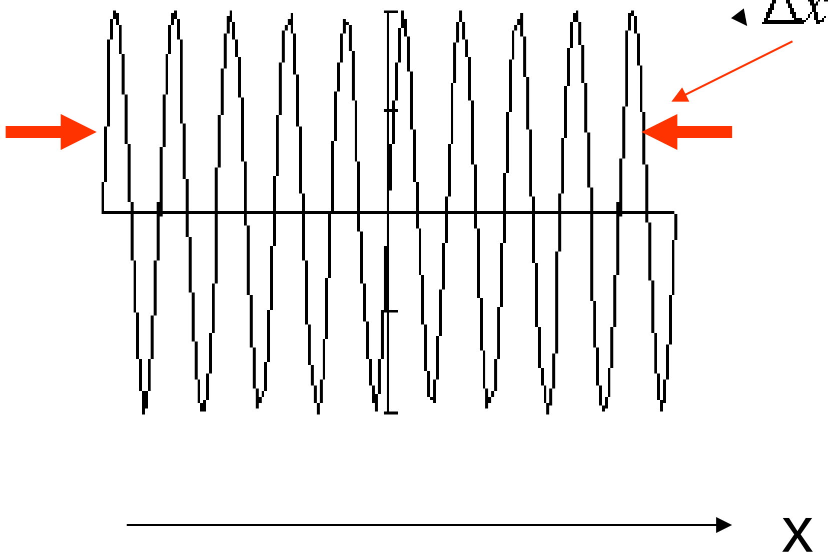
$$A(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} u(x,0) e^{-ikx} dx$$

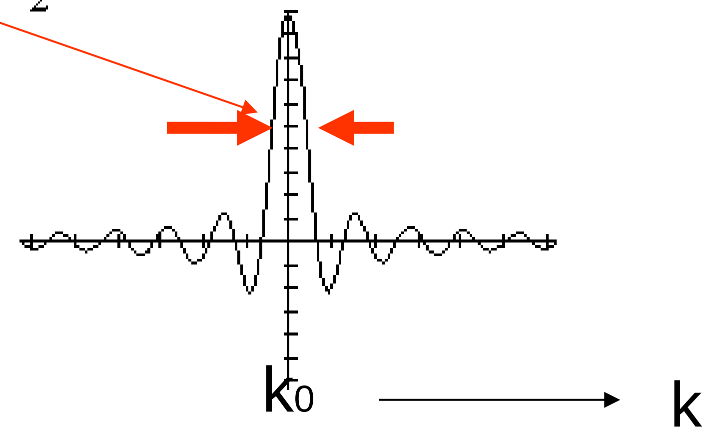
for a fixed wave train:





$$\Delta x \bullet \Delta k \geq \frac{1}{2}$$




$\omega(k)!$ for A(k) relatively narrow concentated around $k_0$

$$\omega(k) = \omega_0 + \frac{d\omega}{dk} \,\Big|_0\, (k - k_0) + \ldots\ldots$$

$$u(k) \approx \frac{e^{i[k_0 (d\omega/dk)|_0 - \omega_0]\cdot t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(k) e^{i[x-(d\omega/dk)\,|_0\, \cdot t]k}\, dk$$

to compare with:

$$0 \leq v_{Phase} \leq \infty$$

that is u(x',0) with

$$x' = x - (d\omega/dk)\,\Big|_0\, \bullet t$$

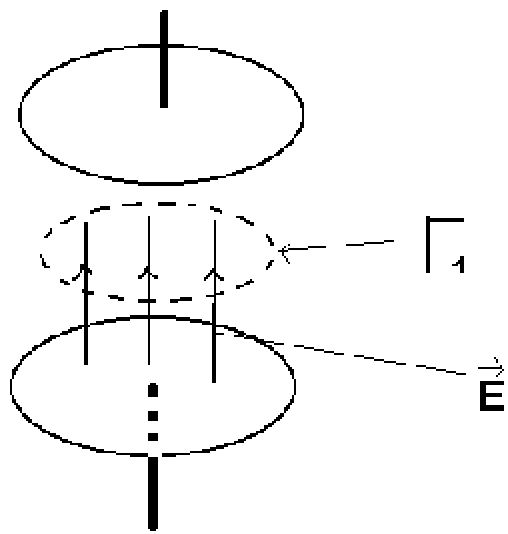
$$\longrightarrow\longrightarrow\longrightarrow v_{group} = (d\omega/dk)\,\Big|_0$$

velocity of the group

$v_{group} \leq c$ always true!

# 10.11. Cavity

Borderline case of a condenser at very high frequencies




$$c^2 \int_{\Gamma_1} \vec{B} \bullet d\vec{s} = \frac{d}{dt} \int \vec{E} \bullet d\vec{A}$$

(area encircled of $\Gamma_1$)

$$c^2 \bullet B \bullet 2\pi r = \frac{d}{dt} E \bullet \pi \bullet r^2$$

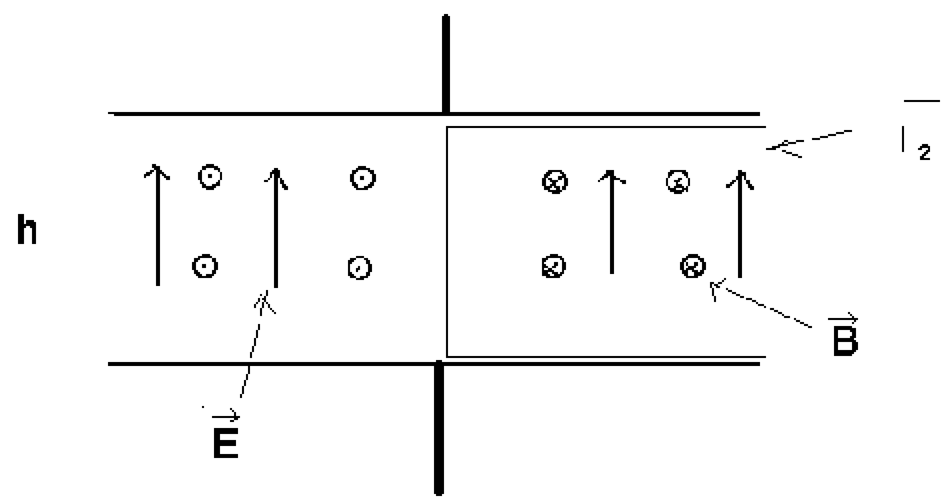
magnetic field in condenser

$$\Rightarrow B = \frac{i\omega r}{2c^2} E_0 e^{i\omega t}$$

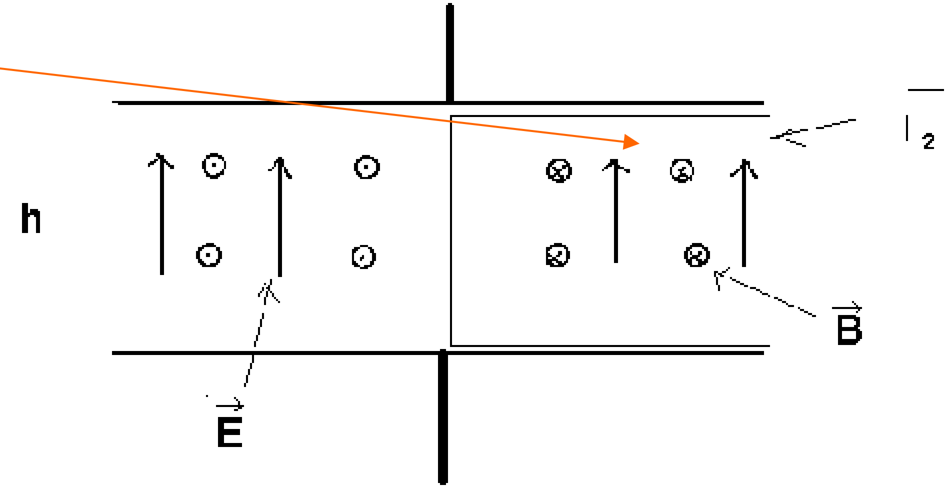
the ',capacity'' is
not anymore $\frac{1}{i\omega C}$

Faraday:

$$\oint_{\Gamma_2} \vec{E} \bullet d\vec{s} = -\frac{d}{dt} \int \vec{B} \bullet d\vec{A}$$



Area encirceld $\Gamma_2$




Aditional contribution $\vec{E}$ :

Separation in two parts

$$E = E_1 + E_2$$

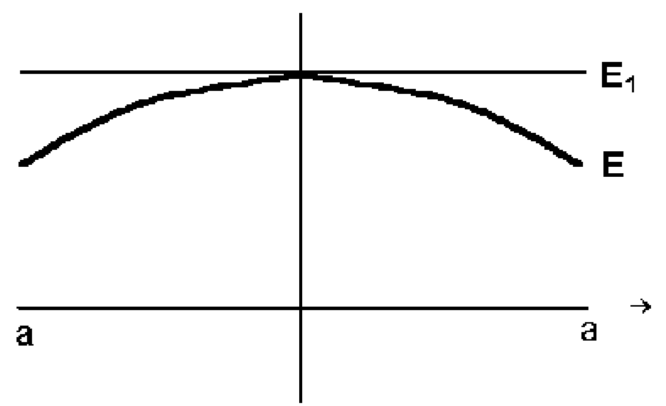
(correction due to B-field) $E_2 = 0$ for r=0 (In center of the condenser)

$$\Rightarrow -E_2 \cdot h = -\frac{d}{dt} h \int \vec{B}(r) \cdot dr$$

$$E_2(r) = \frac{d}{dt} \frac{i\omega \cdot r^2}{4c^2} E_0 e^{i\omega t}$$

$$E_2(r) = -\frac{\omega^2 \cdot r^2}{4c^2} E_0 e^{i\omega t}$$

$$E = E_1 + E_2 = (1 - \frac{1}{4} \frac{\omega^2 r^2}{c^2}) E_0 e^{i\omega t}$$



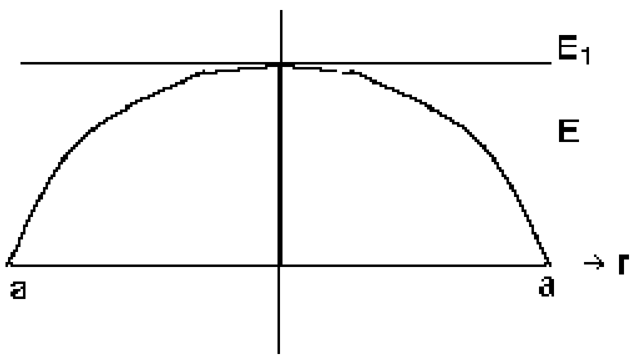
E represents 1. approximation $E_2(t)$ changes too $B$ .........

$$E = E_0 e^{i\omega t}[1 - \frac{1}{(1!)^2}(\frac{\omega\ r}{2c})^2 + \frac{1}{(2!)^2}(\frac{\omega\ r}{2c})^4 - \frac{1}{(3!)^2}(\frac{\omega\ r}{2c})^6 \ldots]$$

Besselfunction: $J_0(x) = 1 - \frac{1}{(1!)^2}(\frac{x}{2})^2 + ..$

( always ansatz of solution at cylinder symmetry)

At even higher frequencies: at $r = 2.405\frac{c}{\omega_0}$

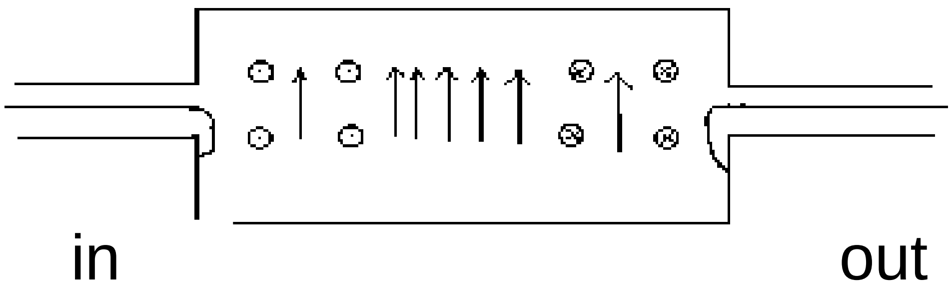



B

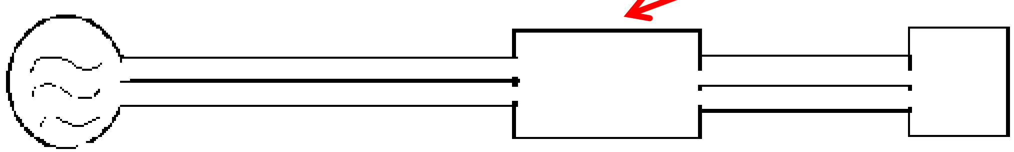

Cavity:

at r=a conducting wall
can be placed!

coupling




high frequency-filter with a resonator:

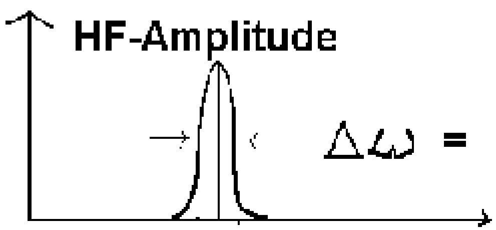



hf-detector

Q=quality of the resonator up to

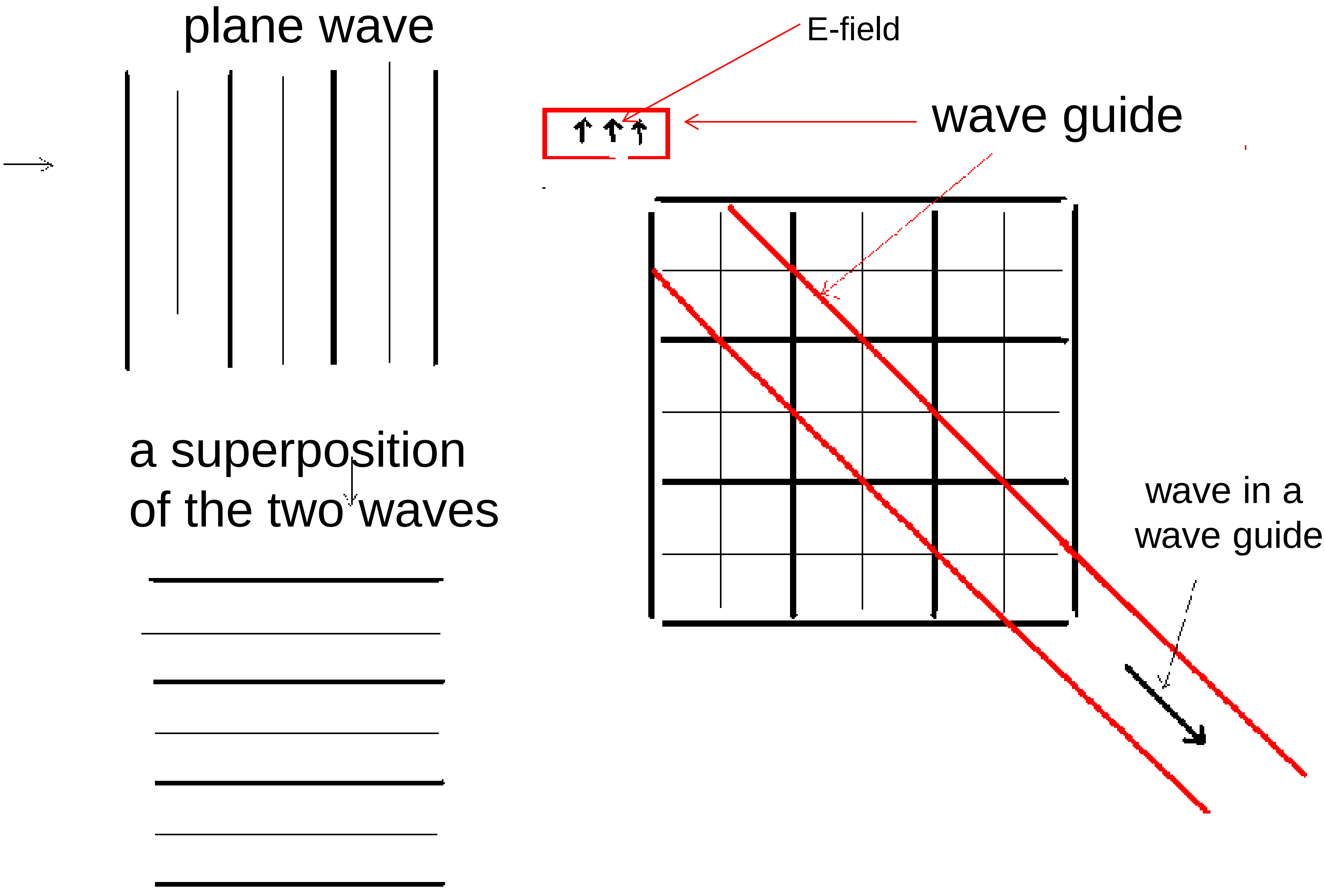
$$10^{10}$$

using super conductivity!




$$\Delta\omega = \frac{\omega_0}{Q}$$

# 10.12 Propagation of waves in rectangular wave guide

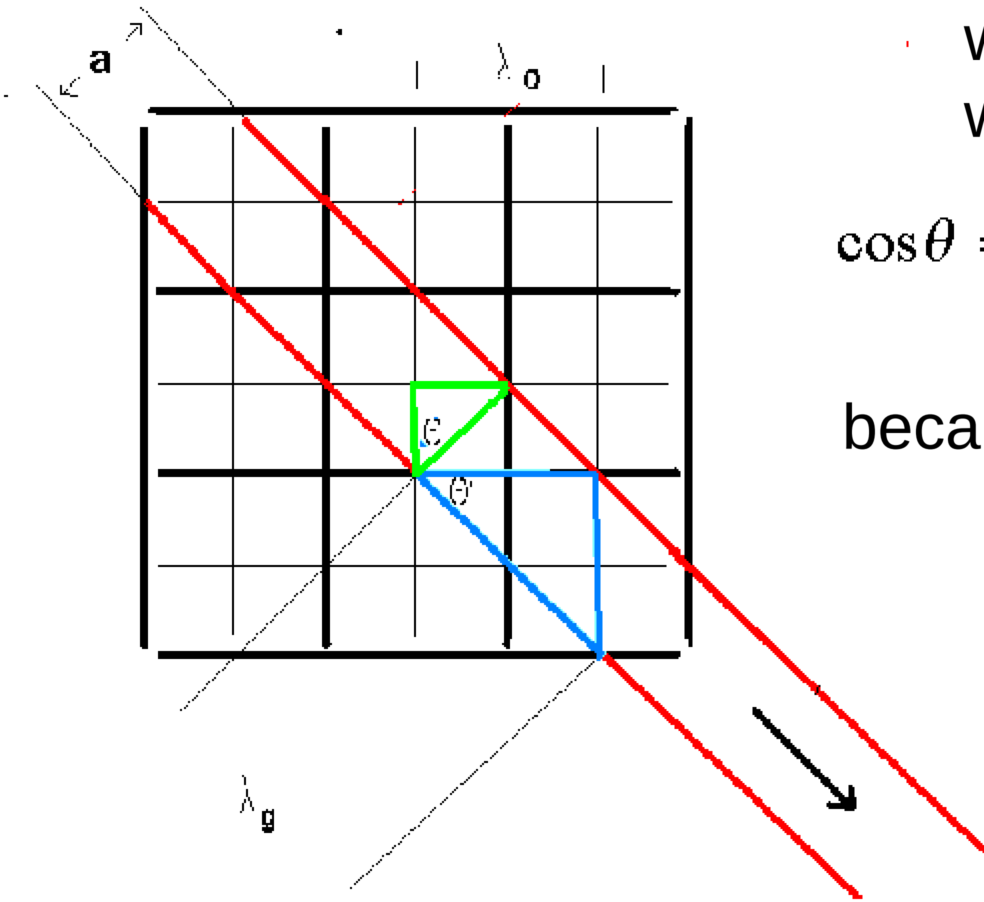


On places with E=0, metal plates can be set

$\Longrightarrow$ wave guide!

$\lambda_0, \lambda_g$ :

wave length without,
wave length in wave guide




$$\cos\theta = \frac{\lambda_0}{\lambda_g} \rightarrow \lambda_g = \frac{\lambda_0}{\cos\theta} = \frac{\lambda_0}{\sqrt{1-(\frac{\lambda_0}{2a})^2}}$$

because $\sin\theta = \frac{\frac{\lambda}{2}}{a} = \frac{\lambda}{2a}$

$\Longrightarrow$

propagation, only if

$$\lambda_0 \prec 2a$$

# beautiful example for phase velocity

always $\succ c$, z be direction of propagaton

$v_{Phase} = \frac{\omega}{k_z} = \lambda_g \bullet \nu$ für $\lambda_0 \to 2a \to v_{Phase} \to \infty$

$$v_{group} = \frac{d\omega}{dk_z} = \frac{d\omega}{dk} \bullet \frac{dk}{dk_z}$$

and $\omega = c \bullet k$

$$k = \frac{2\pi}{\lambda_0}; k_z = \frac{2\pi}{\lambda_g}$$

$$\frac{dk}{dk_z} = \frac{k_z}{k}$$

thus

$$v_{group} = \frac{c^2}{v_{phase}} \prec c \to v_{group} \bullet v_{phase}$$