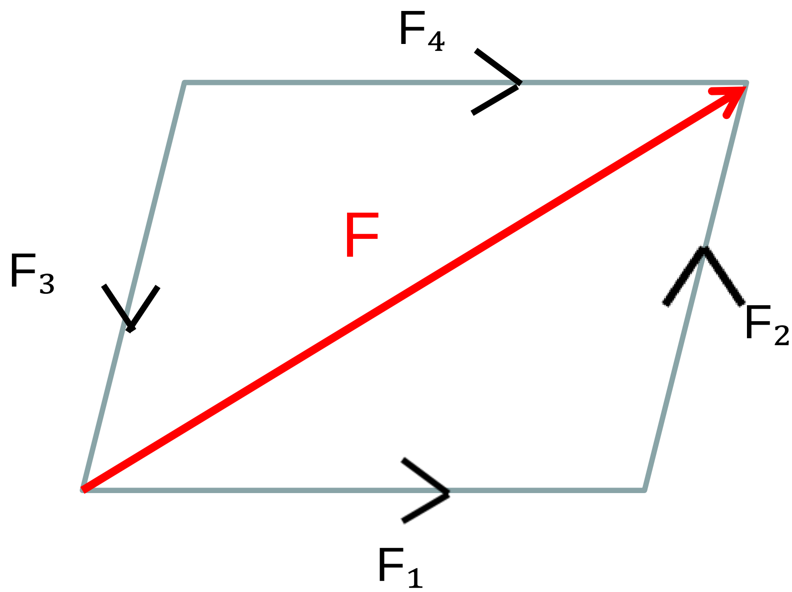
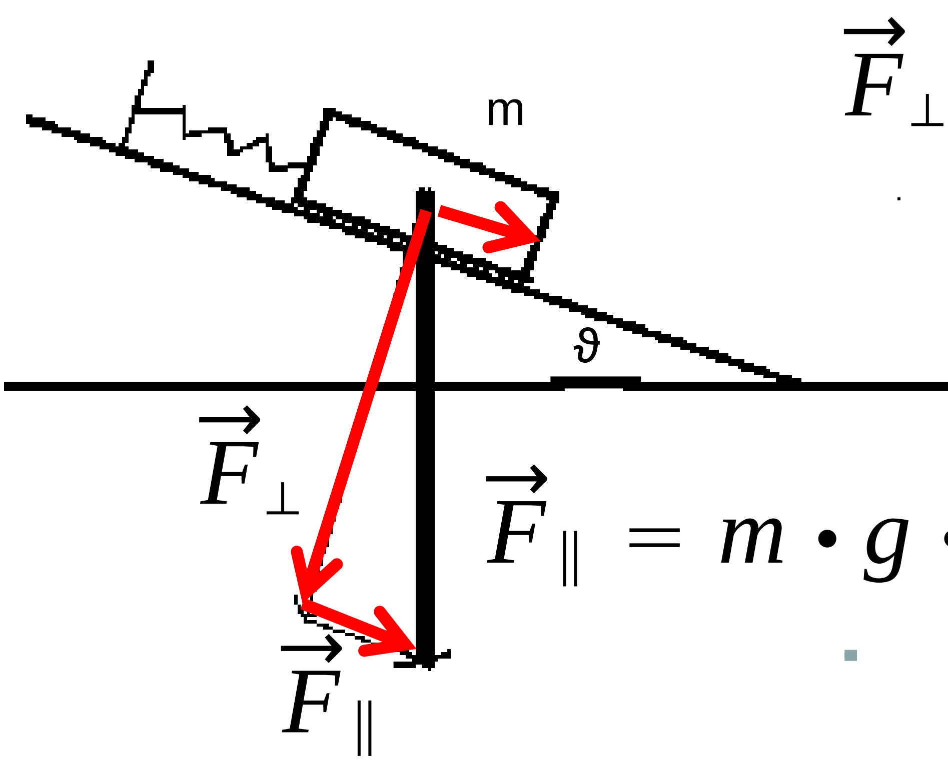
Decomposition of the force into components



Example: Inclined plane



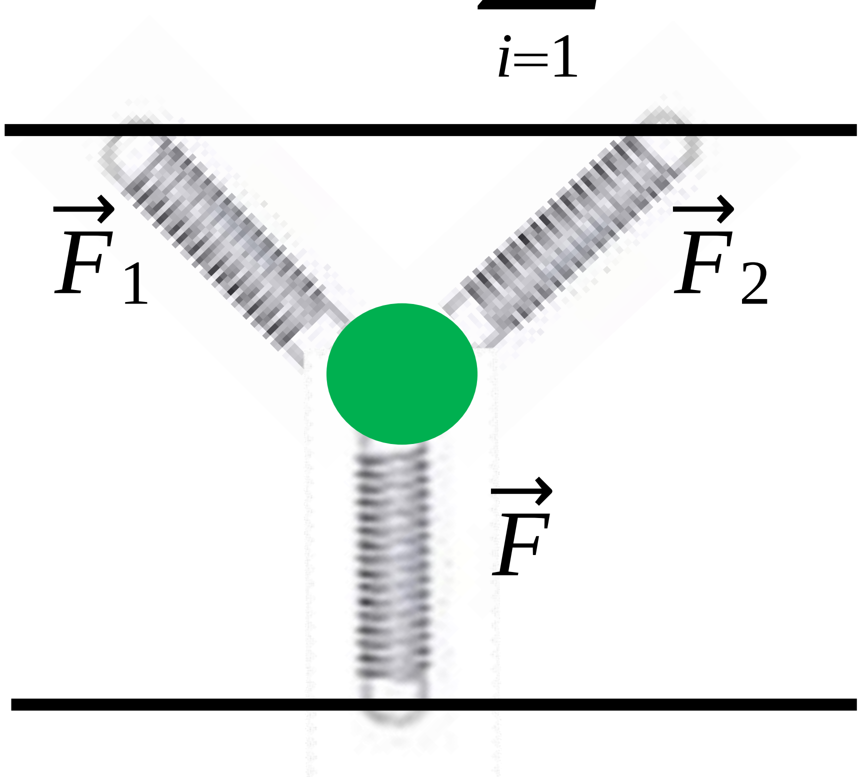
$$\vec{F}_\parallel = m \cdot g \cdot \sin\vartheta$$

$$\vec{F}_{\perp} = m \cdot g \cdot \cos\vartheta$$

For several forces $\vec{F}_i$ the resulting force

$$\vec{F} = \sum_{i=1}^{N} \vec{F}_i$$

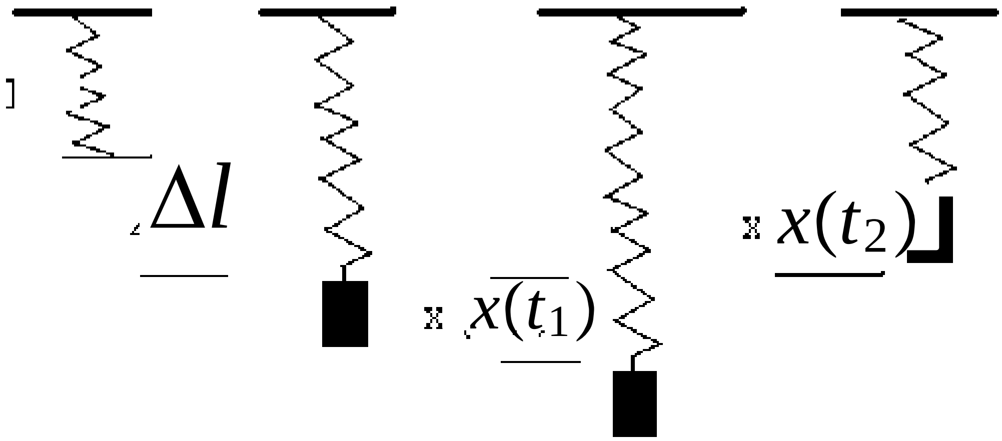
Special case $\vec{F} = \sum_{i=1}^{N} \vec{F}_i = 0$



Is called equilibrium

Force as origin of movement

Example:Elastic pendulum



Equilibrium: Δl Displacements: $x(t_1), x(t_2)$

Additional displacement results in a additional force $\vec{F}$

$$\vec{F} = -D\vec{x}; [F] = N(Newton)$$

Using the second law of Newton:

$$m \cdot \vec{a} = -D \cdot \vec{x}$$

a:Resulting acceleration, m resistance against acceleration

$$m \cdot \ddot{x} = -D \cdot x$$

Linear differential equation

Look for x = x(t)

Result via Ansatz: $x = x_0 \cos \omega t$

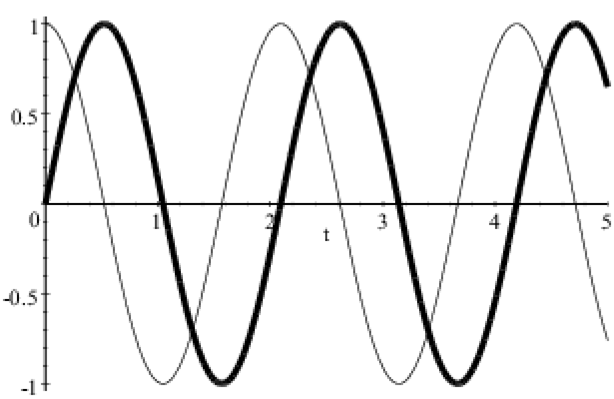
ω = constant $[\omega] = s^{-1}$

Derivation t: $\dot{x} = -\omega x_0 \sin \omega t$

$\ddot{x} = -\omega^2 x_0 \cos \omega t$

$$\rightarrow -m \cdot \omega^2 \cdot x_0 \cdot \cos \omega t = -x_0 \cdot D \cdot \cos \omega t$$

$$\omega^2 = \frac{D}{m} \rightarrow\rightarrow\rightarrow\rightarrow \omega = \sqrt{\frac{D}{m}}$$



Oscillation

Frequency: ν

Period: $\omega = 2\pi \cdot \nu$

T: Time of the period

$$\omega = 2\pi\nu = \frac{2\pi}{T} = \frac{\text{angle of the full circle}}{\text{running time of the circumference}}$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{D}}$$

Consequence: Starting with the assumption of a linear force yields a harmonic oscillation.
T independent of the amplitude.

1.3 Work and Power

$$work = force \cdot distance$$

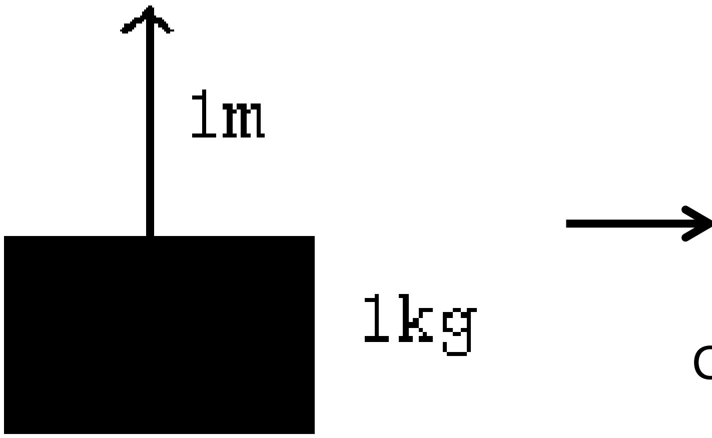
$$w = \vec{F} \cdot \vec{s}$$

Scalar (W) = Joule =

$$\mathrm{N \cdot m = W \cdot s}$$

Connection with Electrodynamics:
Watt * second

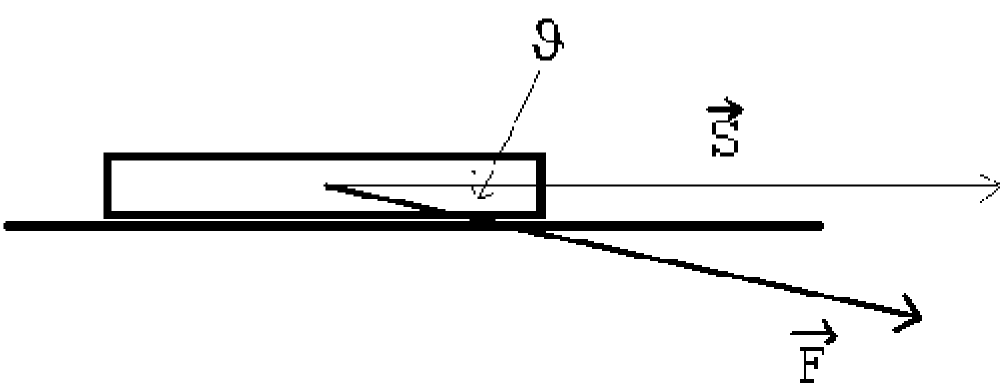
Work against gravity



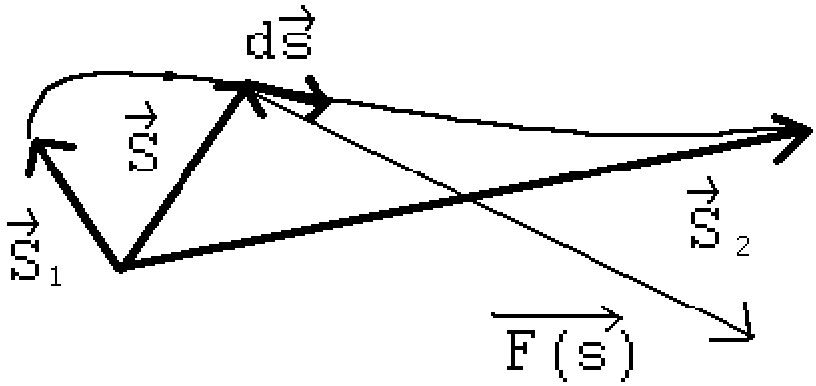
9.81 Joule
Ws
Nm

Compare:
2.35 cal
cal = 4.18Nm
Calorie:
Unit in Thermo-dynamics

Scalar product of two vectors:



Here work done on a distance S



$$dW = \vec{F}(\vec{s})d\vec{s} \rightarrow W = \int_{S_1}^{S_2} \vec{F}(\vec{s})d\vec{s}$$

Power $\mathrm{P} = \quad = \frac{dW}{dt} = \dot{W}; [P] = J \cdot s^{-1} = Watt$

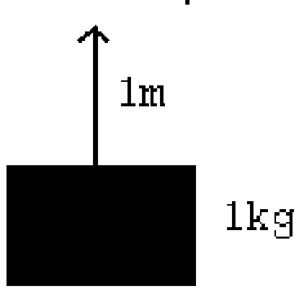
$$\mathrm{P} = \frac{\vec{F}(t) \cdot d\vec{s}}{dt} = \vec{F}(t) \cdot \vec{v}(t)$$

1.4 Kinetic and potential energy

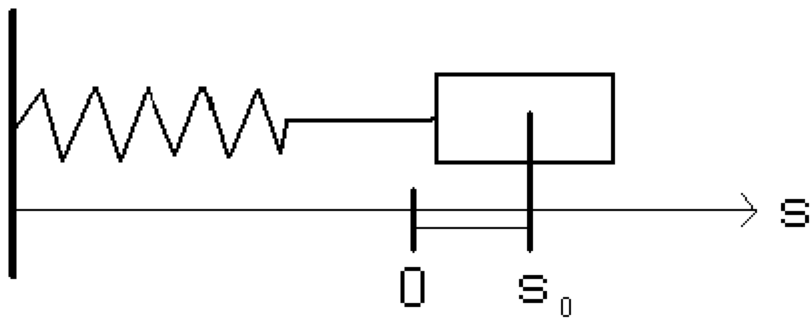
a) Potential Energy

Within the earth gravitational field: $m \cdot g \cdot h$

The work done remains as potential energy available: e.g,;.



Another example: Potential energy stored in a string

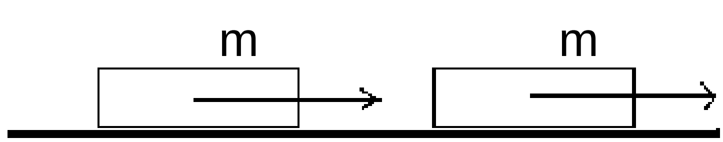


⟵ $-D \cdot s_0$

$$E_{pot} = \int_0^{s_0} D \cdot s \cdot ds = \frac{D \cdot s_0^2}{2}$$

b) Kinetic energy

Force: Reason of movement



t=0	t
s=0	s
v=0	v

Between s=0 and s:

Work done: $W = F \cdot s$

Energy must be due to movement of the body

$$F = a \cdot m = \frac{dv}{dt} \cdot m = m \cdot \frac{dv}{ds} \cdot \frac{ds}{dt} = m \cdot v \cdot \frac{dv}{ds}$$

$$F \cdot ds = m \cdot v \cdot dv$$

$$\int_0^s F \cdot ds = dW = \int_0^v m \cdot v' \cdot dv'$$

Constant force ➞ $$F \cdot \int_0^s ds = m \cdot \int_0^v v' \cdot dv'$$

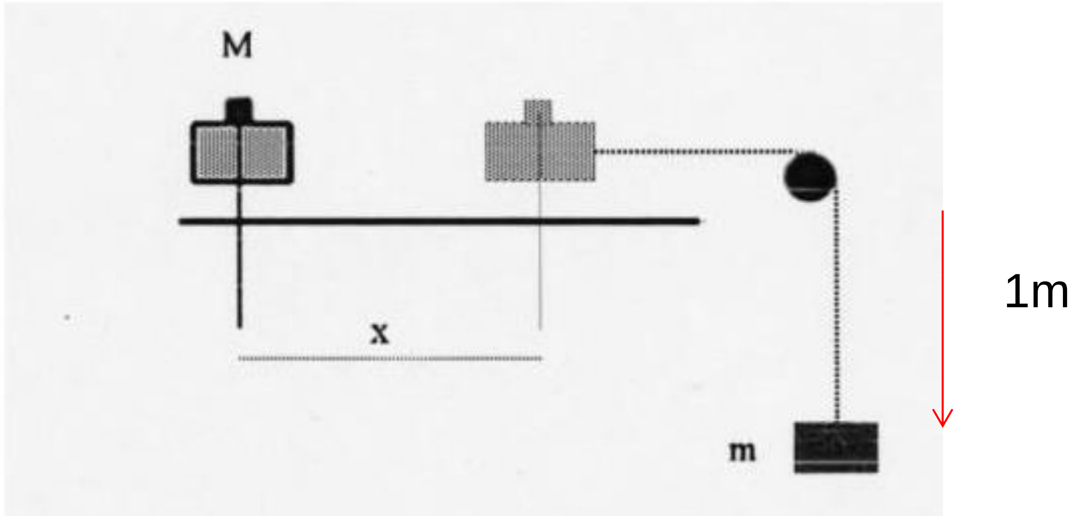
$$F \cdot s = \frac{m}{2} v^2$$

Work gets kinetic enery

$$E_{kin} = \frac{1}{2} m \cdot v^2$$

Experiment in a gravitational field

$$M=200g, m=10g$$



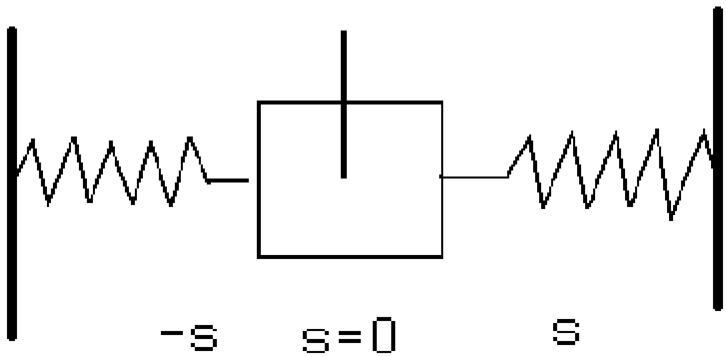
$x(m)$	Δt	v	v^2	$0.21 \cdot v^2$
1	1.065	0.939	0.882	0.0926

To compare with:

$$m \cdot g \cdot x = 0.01 \cdot 9.81 \cdot 1 = 0.0981 \ (Nm)$$

Discrepancy: Friction

Experiment: Energy stored in a string :



S= Δx

$$\mathrm{E}_{kin} = \frac{m}{2}\left(\frac{\Delta x}{\Delta t}\right)^2 \qquad E_{pot} = \frac{1}{2} D \cdot s^2$$

Δt (s)	$\pm\ \Delta x$ (m)	m (kg)	E_{kin} Joule
0.277	0.01	0.205	0.013

Extracted from oscillation time

$$T = 2\pi\sqrt{\frac{m}{D}} = 1.7s$$

$$D = 2.8kg \cdot s^{-2}$$ and s=0.01m

$$\mathrm{E}_{pot} = 0.014J$$

Conclusion: Conservation of energy:

In a closed system :

$$E_{kin} + E_{pot} = \text{constant}$$

Power: s.above.

$$\mathrm{P} = \frac{\vec{F}(t) \cdot d\vec{s}}{dt} = \vec{F}(t) \cdot \vec{v}(t)$$

Examples for magnitudes

Radiation of sun on top of earts atmospere:: 1.367 kW/qm
Received on surface ca 1 kW/qm
Powerplant: $1\ \text{GW} = 10^9\, Watt$

Car: e.g.. : 50 kW

Plate on oven: 2 kW

Further examples:

Person: 80kg climps in 10s on a staircase of 5m;

$$\Longrightarrow P = \frac{W}{t} = \overrightarrow{F} \cdot \overrightarrow{S}/t = m \cdot \overrightarrow{g} \cdot \overrightarrow{S}/t$$

$$= \frac{80 \cdot 9.81 \cdot 5}{10} = 392.4 N \cdot m/s = 0.53 PS$$

Mass m starting with constant power accelerating: (e.g.: E-loc.

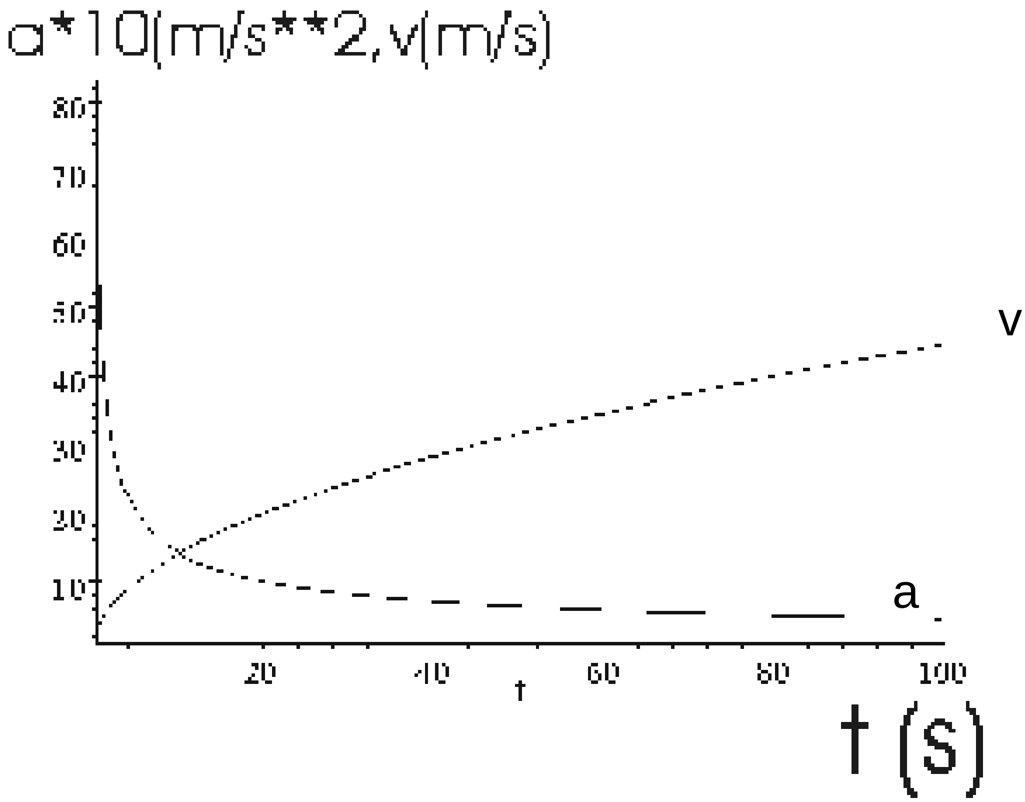
$$: \overrightarrow{F} \mid\mid \overrightarrow{v} \quad v?, a?$$

$$F = \frac{P}{v}; F = m \cdot a \Longrightarrow \frac{P}{v} = m \cdot \frac{dv}{dt}$$

$$\Longrightarrow v \cdot dv = \frac{P}{m} dt \Longrightarrow$$

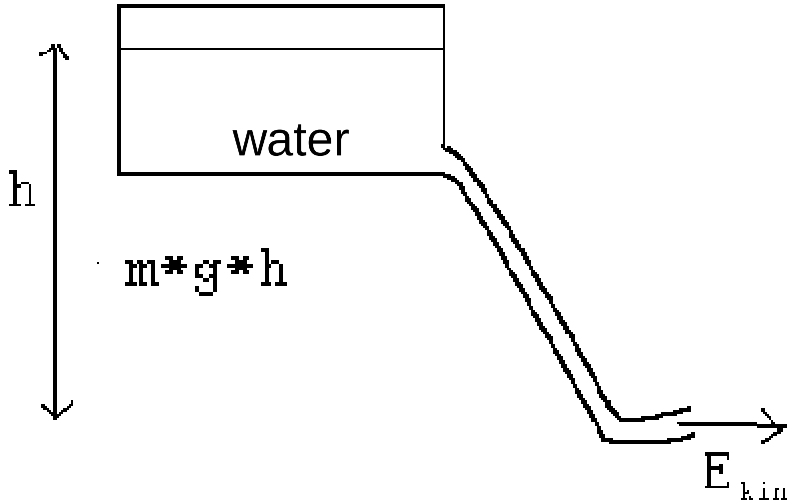
$$\int_0^{v(t)} v \cdot dv = \frac{P}{m} \cdot \int_0^t dt' \Longrightarrow \quad v^2(t) = \frac{2 \cdot P \cdot t}{m} \Longrightarrow$$

$$v(t) = \sqrt{\frac{2 \cdot P \cdot t}{m}} \qquad a(t) = \sqrt{\frac{0.5 \cdot P}{m \cdot t}}$$



Dam: Convert potential energy to kin. energy

$$v=\sqrt{2 \cdot g \cdot h}$$



Kaprun: h=800m

v=125m/s=
451km/h

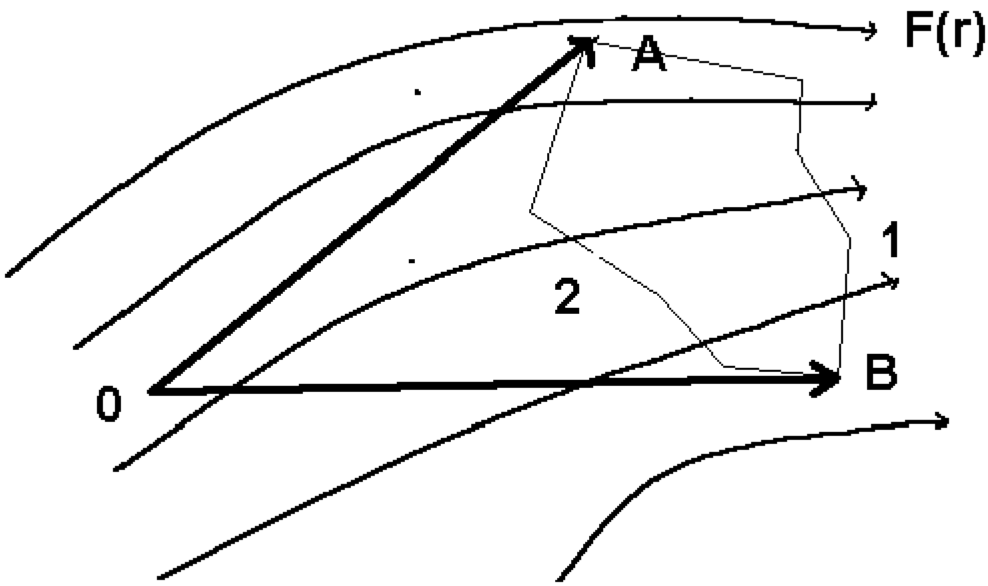
1.5 Field of forces, Potential

Field of forces: Region, a mass point feel s a force

$$\vec{F} = \vec{F}(\vec{r})$$

Mass point transfer from A to B

$\rightarrow$ Work: $W(r_1, r_2) = \int_A^B \vec{F}(\vec{r}) d\vec{r}$



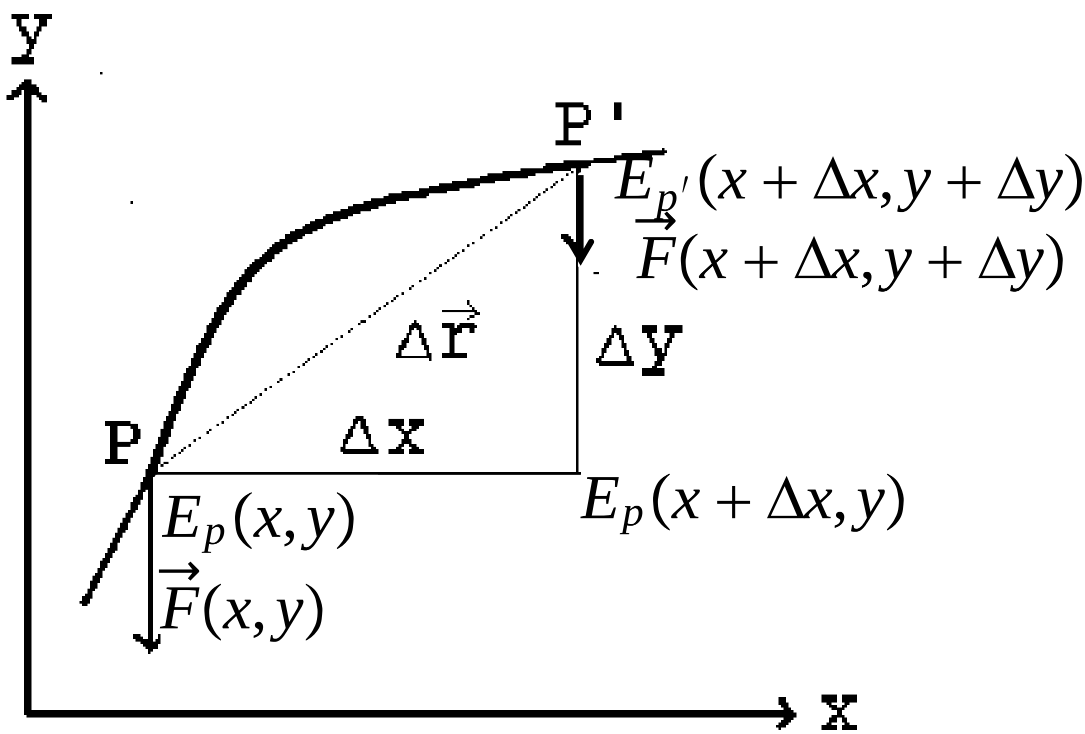
In many cases $W(r_1, r_2)$ independent of course

One speaks of conservative forces (fields).

$$\Longrightarrow \int_A^B \overrightarrow{F}(\overrightarrow{r})d\overrightarrow{r}\ \text{course1}\ = \int_A^B \overrightarrow{F}(\overrightarrow{r})d\overrightarrow{r}\ \text{course2}$$

$$\oint \overrightarrow{F}(\overrightarrow{r})d\overrightarrow{r} = 0$$

In a conservative force field:



E_p Potential energy

$$\mathrm{E}_p(x,y) \rightarrow \mathrm{E}_p(x+\Delta x, y+\Delta y)$$

$$\mathrm{E}_p(x+\Delta x, y+\Delta y) - \mathrm{E}_p(x,y) = \Delta \mathrm{E}_p = \frac{\partial E_p}{\partial x}\Delta x + \frac{\partial E_p}{\partial y}\Delta y$$

$P' \rightarrow\rightarrow P$ Change of potential energy

P' $\longrightarrow\longrightarrow\longrightarrow$ P

$$\Delta W = \overrightarrow{F} \cdot \Delta \overrightarrow{r} = -\Delta \mathrm{E}_p \longrightarrow\longrightarrow$$

$$F_x \cdot \Delta x + F_y \cdot \Delta y = -\frac{\partial E_p}{\partial x}\Delta x - \frac{\partial E_p}{\partial y}\Delta y$$

$$\longrightarrow\longrightarrow\longrightarrow F_x = -\frac{\partial E_p}{\partial x}, F_y = -\frac{\partial E_p}{\partial y}$$

In general:

$$\boxed{F = -grad\ E_p = -\nabla E_p}$$

A look forward towards theoretical mechanics

$$m\ddot{x} - F(x,t) = 0$$

$$m\ddot{x} + \frac{\partial E_p}{\partial x} = 0$$

Newton

$$\frac{d}{dt}\frac{\partial}{\partial \dot{x}}\left(\frac{m\dot{x}^2}{2}\right) = \frac{d}{dt}\frac{\partial E_{kin}}{\partial \dot{x}}$$

$$\frac{d}{dt}\frac{\partial E_{kin}}{\partial \dot{x}} + \frac{\partial E_p}{\partial x} = 0$$

mit $\frac{\partial L}{\partial \dot{x}} = \frac{\partial E_{kin}}{\partial \dot{x}}$ und $\frac{\partial L}{\partial x} = -\frac{\partial E_p}{\partial x}$

Lagrange Function: $L(\dot{x}, x) = E_{kin}(\dot{x}) - E_p(x)$

The law of Newton can be written as:

$$\frac{d}{dt}\frac{\partial L(x,\dot{x})}{\partial \dot{x}} - \frac{\partial L(x,\dot{x})}{\partial x} = 0$$

Feynman II,19

That is,how nature works!