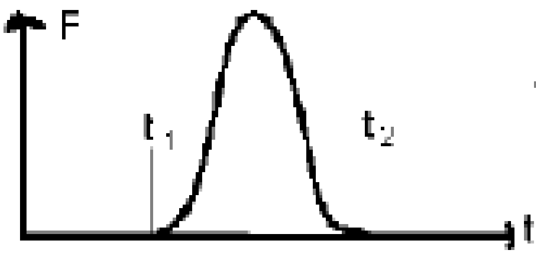
b) Elastic collision

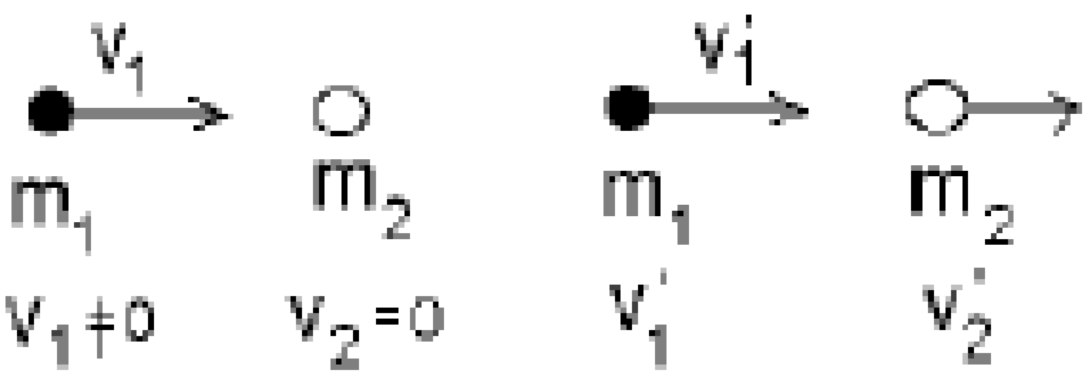


$$\int_{t_1}^{t_2} \vec{F} \cdot dt = \vec{p}(t_2) - \vec{p}(t_1) \; :$$

Change of momentum

Elastic means: Collision process does not consume energy

$$E_{kin}(prior) = E_{kin}(after)$$



Collision in a straight line

Conservation of momentum of energy:

$$m_1 \cdot v_1 = m_1 \cdot v_1' + m_2 \cdot v_2' \quad 1$$

$$\frac{1}{2} m_1 \cdot v_1^2 = \frac{1}{2} m_1 \cdot v_1'^2 + \frac{1}{2} \cdot m_2 \cdot v_2'^2 \quad 2$$

From 1 $$m_1^2 \cdot (v_1 - v_1')^2 = m_2^2 \cdot v_2'^2 \quad 3$$

From 2 $$m_2 \cdot m_1 \cdot (v_1^2 - v_1'^2) = m_2^2 \cdot v_2'^2 \quad 4$$

$$\frac{4}{3} \rightarrow\rightarrow\rightarrow \frac{m_2 \cdot (v_1 + v_1')}{m_1 \cdot (v_1 - v_1')} = 1$$

$$\rightarrow\rightarrow\rightarrow\rightarrow m_2 \cdot v_1 + m_2 \cdot v_1' = m_1 \cdot v_1 - m_1 \cdot v_1'$$

$$v_1' \cdot (m_2 + m_1) = (m_1 - m_2) \cdot v_1$$

Velocity of the pushing body after the collision

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1$$

v'_2 ?

$$m_1 \cdot v_1 = m_1 \cdot v_1' + m_2 \cdot v_2' \qquad m_1 \cdot v_1 - m_1 \cdot \frac{m_1 - m_2}{m_1 + m_2} v_1 = m_2 \cdot v_2'$$

$$v_2' = \frac{2 \cdot m_1}{m_1 + m_2} v_1$$

Different cases

$m_1 = m_2$

v	p transfer	E transfer
v'(2)=v(1)	\ complete	complete
v'(1)=0	p(1)->p(2)	E(1)->E(2)'

$m_1 \ll m_2$

v'(2)<<v(1)	Max.p -transfer	Min.E- -transfer
v'(1)~-v(1)	2p(1)	V(2)'~0

Like a reflection on a wall

$m_1 \gg m_2$

v	p	E
v(1)'~v(1)	small	small
v(2)'~2v(1)!	p(1)'-p(1)~0	

c) Inelastic collision

Mechanical energy gets lost

c.m,-momentum conserved:

$$m_1 \cdot v_1 = (m_1 + m_2) \cdot v_{SP} \quad \rightarrow v_{SP} = \frac{m_1 \cdot v_1}{m_1 + m_2}$$

Energy Q gets lost

$$Q=\frac{1}{2}m_1\cdot v_1^2-\frac{1}{2}(m_1+m_2)\cdot v_{SP}^2 \qquad v_{SP}=\frac{m_1\cdot v_1}{m_1+m_2}$$

$$\frac{1}{2}m_1\cdot v_1^2-\frac{1}{2}(m_1+m_2)\cdot\frac{m_1^2}{(m_1+m_2)^2}v_1^2$$

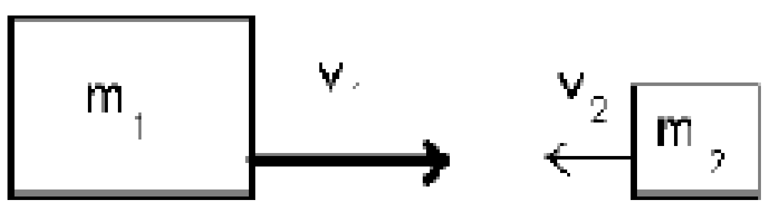
$$\frac{1}{2}m_1\cdot v_1^2\left(1-\frac{m_1}{m_1+m_2}\right)=\frac{1}{2}m_1\cdot v_1^2\cdot\frac{m_2}{m_1+m_2}$$

$$m_1\ll m_2; Q=E_{kin};$$

$$m_1=m_2 \longrightarrow Q=\frac{E_{kin}}{2}$$

Complete conversion

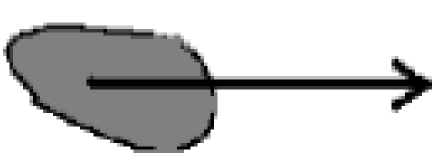
Example:Crash of two cars



Prior $v_1=v_2$ $m_1=4\cdot m_2$

c.m.:? $p=m_1\cdot v_1-m_2\cdot v_1=(m_1+m_2)\cdot v_{SP}$
$=v_1\cdot(m_1-m_2)$

After



$$v_{SP}=\frac{v_1\cdot(m_1-m_2)}{m_1+m_2}$$

inelastic

Change of velocity

$$m_1 : \Delta v_1 = \frac{2}{5} v_1$$

$$m_2 : \Delta v_2 = \frac{8}{5} v_1$$

With: $v_1 = 50 km/h$

Catapulted back

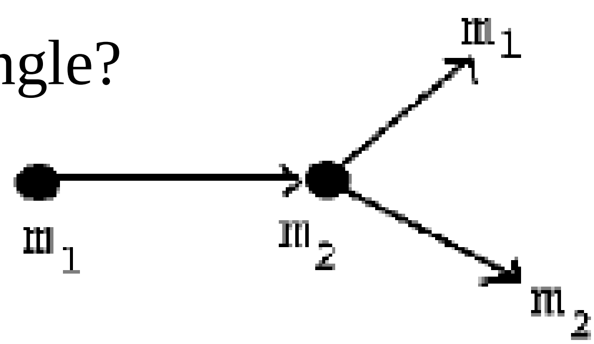
$\Delta v_1 = 20 km/h \quad \Delta v_2 = 80 km/h$

Injuries by acceleration!!

Important $\frac{\Delta v}{\Delta t}$ ~ Ratio of masses

Next example: Elastic collision in a plane

What is the scattering angle?



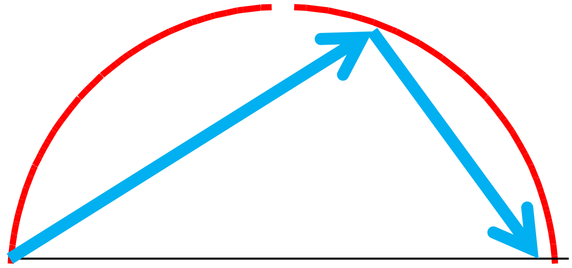
Special case:

$m_1 = m_2 = m, v_2 = 0$

Conservation of energy:

$$\frac{p_1^2}{2m} = \frac{p_1'^2}{2m} + \frac{p_2'^2}{2m}$$

$$\rightarrow\rightarrow\rightarrow\rightarrow\rightarrow \; p_1^2 = p_1'^2 + p_2'^2$$



After scattering:
The bodies fly apart
with an angle of 90 deg.