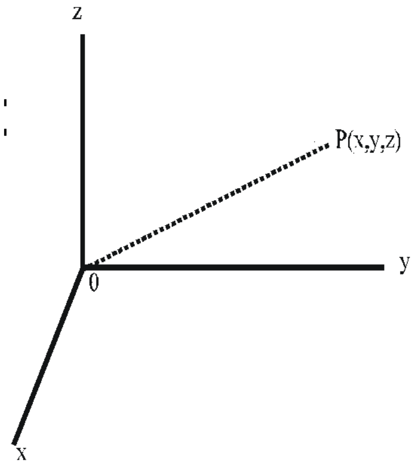
1.7 Frame of reference and forces of inertia

Physical quantities are details of measured variables

e.g.:location:



$$\vec{r} = x\vec{e}_x + y\vec{e}_y + z\vec{e}_z$$

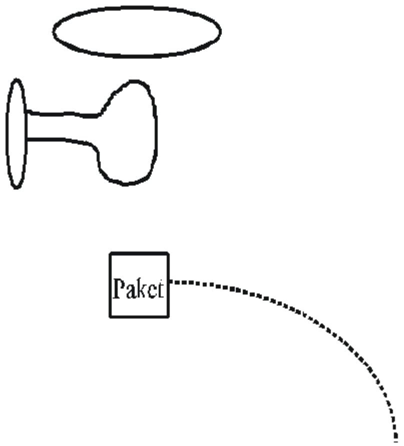
Fixing sero points
classical mechanics

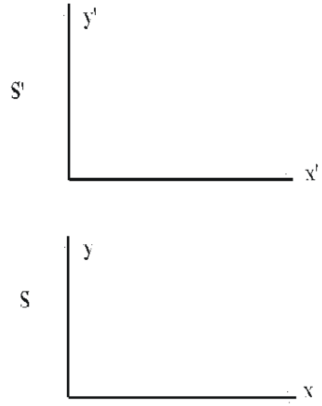
a) Point sero of time and time scale are independent of position and other properties

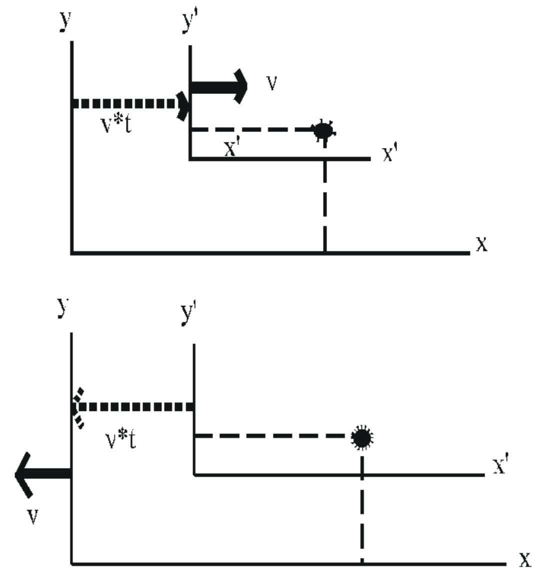
b) Scales of position measurements are equal in reference systems moving relatively to each other.

Examples for reference systems:
center of earth, rotation axis of earth.

Example: Helicopter (Velocity v) provides a climber with a parcel





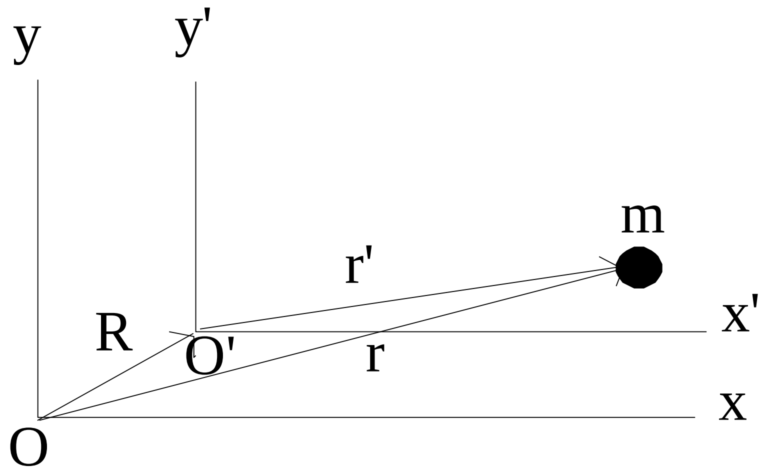


The parcel approaches the climber on a parabola in the climbers system.

The climber sees y‘ arriving.

The pilot of the helicopter sees the parcel fall in in a straight line beeing in the y‘-system

1.7.1 moving in a system of reference



m= moves due to

$$\vec{F} = m\vec{a} = m\,\ddot{\vec{r}}$$

movement in

$$\Sigma(0;x,y,z)$$

$\Sigma'(0;x',y')$ Moves with v relatively to Σ

Position vector:

$$\vec{r}\,'(t) = \vec{r}(t) - \vec{R}(t)$$

Velocity:

$$\dot{\vec{r}}\,'(t) = \dot{\vec{r}}(t) - \dot{\vec{R}} = \vec{v}(t) - \vec{v}_0$$

Acceleration:

$$\vec{a}\,'(t) = \frac{d}{dt}(\vec{v}(t) - \vec{v}_0) = \frac{d}{dt}\vec{v}(t) = \vec{a}(t)$$

$$\boxed{\vec{F} = m\vec{a} = m\vec{a}\,'}$$

As long as the coordinate systems move against each other with v=const.

Effect due to F independent in which system a will be measured!

1.7.2. Uniform accelerated reference system

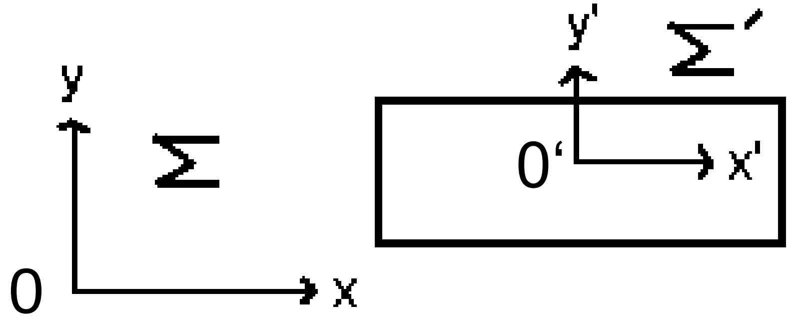
Example : Starting or a slowdown rail

System Σ'

gets

accelerated

relatively to: Σ



$$\vec{a}_0 = \overrightarrow{\text{constant}}$$

For t=0; 0'=0

Relative velocity:

Position vector:

$$\vec{v}_0(t) = \vec{a}_0 \cdot t; \vec{R}(t) = \frac{1}{2}\vec{a}_0 \cdot t^2$$

$$\vec{r}\,'(t) = \vec{r}(t) - \vec{R}(t) = \vec{r}(t) - \frac{1}{2}\vec{a}_0 \cdot t^2$$

Velocity:

$$\vec{v}\,'(t) = \frac{d}{dt}\vec{r}\,'(t) = \vec{v}(t) - \vec{a}_0 \cdot t$$

Acceleration:

$$\vec{a}\,'(t) = \vec{a}(t) - \vec{a}_0$$

An outside force $\vec{F}$ may in Σ create a acceleration $\vec{a}$

$$\vec{F} = m \bullet \vec{a}$$

In Σ' : $\vec{a}' \rightarrow m \bullet \vec{a}' = m \bullet (\vec{a} - \vec{a}_0) = \vec{F} - m \bullet \vec{a}_0 = \vec{F}'$

Additional force due to an accelerated relative movement **"Pseudo-force"**

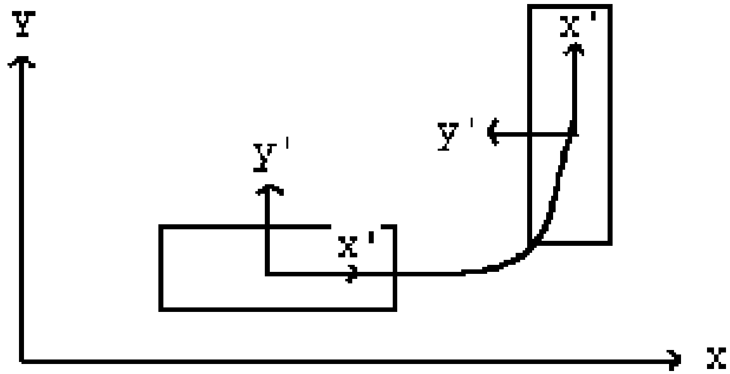
By a proper choice of a: $\vec{a}_0 \rightarrow \vec{F}' = 0,$

no acceleration in Σ' : e.g.: free fall: $\vec{a}_0 = \vec{g}$

1.7.3. Rotational reference systems

Example:

A car moves along a curve



Passenger feels a force,

which drives him out of the path .

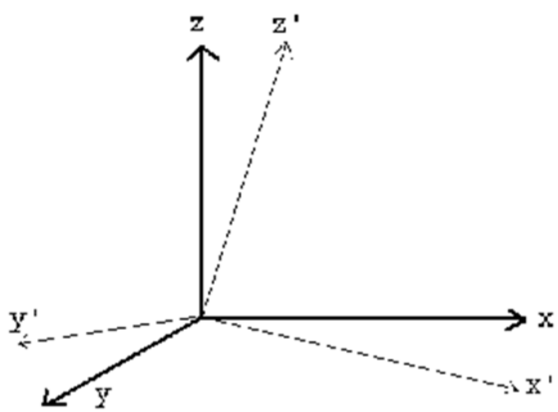
Σ and Σ'

$$\vec{r} = \vec{r}'$$

$$x(t)\cdot\vec{e}_x + y(t)\cdot\vec{e}_y + z(t)\cdot\vec{e}_z = x(t)'\cdot\vec{e}_{x'} + y(t)'\cdot\vec{e}_{y'} + z(t)'\cdot\vec{e}_{z'}$$

$$\vec{v} = \dot{x}\cdot\vec{e}_x + \dot{y}\cdot\vec{e}_y + \dot{z}\cdot\vec{e}_z = \dot{x}'\cdot\vec{e}_{x'} + \dot{y}'\cdot\vec{e}_{y'} + \dot{z}'\cdot\vec{e}_{z'} +$$

$$x'\cdot\dot{\vec{e}}_{x'} + y'\cdot\dot{\vec{e}}_{y'} + z'\cdot\dot{\vec{e}}_{z'}$$

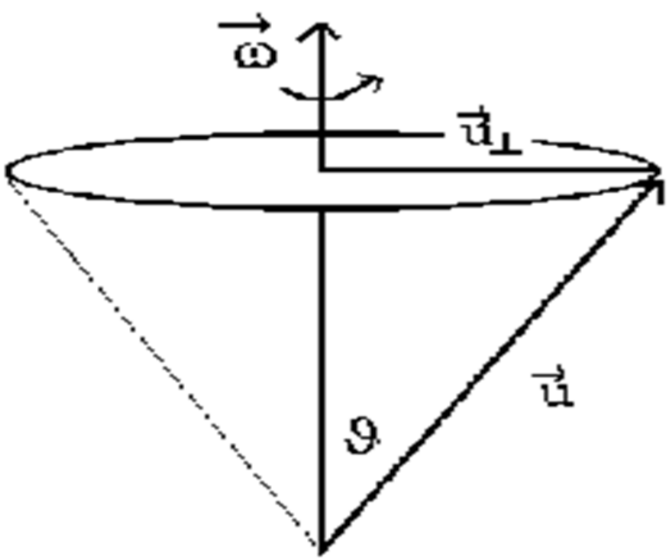


$$\vec{a} = \ddot{x}' \cdot \vec{e}_{x'} + \ddot{y}' \cdot \vec{e}_{y'} + \ddot{z}' \cdot \vec{e}_{z'} + 2 \cdot \left[\dot{x}' \cdot \dot{\vec{e}}_{x'} + \dot{y}' \cdot \dot{\vec{e}}_{y'} + \dot{z}' \cdot \dot{\vec{e}}_{z'} \right] + x' \cdot \ddot{\vec{e}}_{x'} + y' \cdot \ddot{\vec{e}}_{y'} + z' \cdot \ddot{\vec{e}}_{z'}$$

Acceleration due to pseudo forces

(derivatives of the unit vectors)

Rotation of vectors: $\vec{u}$ around its axis of rotation $\vec{\omega}$



$\frac{d\vec{u}}{dt} \perp$ on $\vec{u}$ and $\vec{\omega}$

$$\left| \frac{d\vec{u}}{dt} \right| = \omega \cdot u_\perp = \omega \cdot u \cdot \sin\vartheta$$

According to the definition of the products of vectors: $\frac{d\vec{u}}{dt} = \vec{\omega} \times \vec{u}$

For $\vec{u}$ with unit vectors $\vec{e}_{x'}, \vec{e}_{y'}, \vec{e}_{z'} \rightarrow \dot{\vec{e}}_{x'} = \vec{\omega} \times \vec{e}_{x'}$

$$\dot{\vec{e}}_{y'} = \vec{\omega} \times \vec{e}_{y'}$$

$$\dot{\vec{e}}_{z'} = \vec{\omega} \times \vec{e}_{z'}$$

$$\rightarrow \quad \ddot{\vec{e}}_{x'} = \vec{\omega} \times \dot{\vec{e}}_{x'} = \vec{\omega} \times \left(\vec{\omega} \times \vec{e}_{x'} \right)$$

$$\rightarrow \vec{a} = \vec{a}' + 2 \cdot \omega \times \left[\dot{x}' \cdot \vec{e}_{x'} + \dot{y}' \cdot \vec{e}_{y'} + \dot{z}' \cdot \vec{e}_{z'} \right] +$$

$$\vec{\omega} \times [\vec{\omega} \times (x(t)' \cdot \vec{e}_{x'} + y(t)' \cdot \vec{e}_{y'} + z(t)' \cdot \vec{e}_{z'})]$$

$$\boxed{\vec{\omega} \times [\vec{\omega} \times (x(t)' \cdot \vec{e}_{x'} + y(t)' \cdot \vec{e}_{y'} + z(t)' \cdot \vec{e}_{z'})]}$$

(centrifugal force) *on m*

Σ: $\vec{F}$ acts on m

$$\Sigma' : \vec{F}' = \vec{F} - 2 \cdot \vec{\omega} \times \vec{v}' \cdot m - \vec{\omega} \times \left(\vec{\omega} \times \vec{r}' \right) \cdot m$$

$\vec{F}'_C$

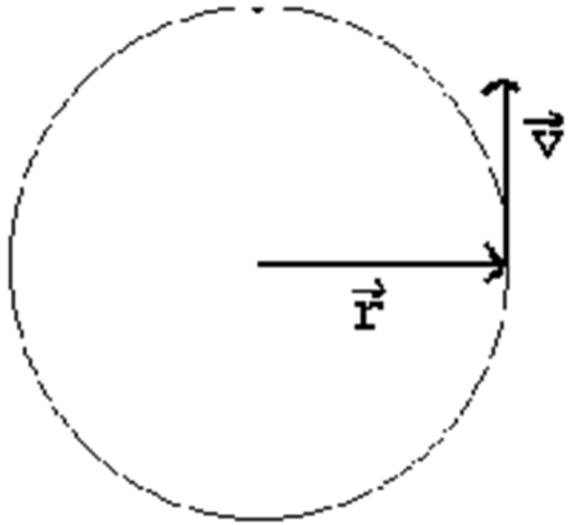
Coriolis force

Centrifugal force

Pseudo-force

Discussion: Uniform motions of a mass point

$\omega = \frac{v}{r}$ =constant



Σ : $\vec{F}_r$: **Zentripetal force**

$a_r = \omega^2 \cdot r \rightarrow\rightarrow\rightarrow$

towards center, forces m
on a circular orbit (e.g. with thread)!

v=constant,
m is at rest

In Σ', which rotates with ω

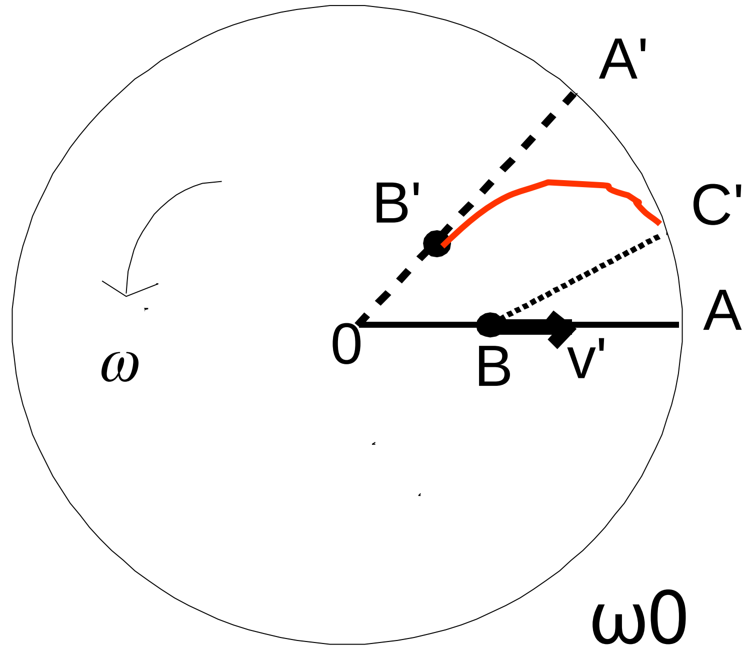
$$\rightarrow F' = 0 = \vec{F}_r + \vec{F}_N$$

Zentripetal force =*Zentrifugal force (Pseudo force)*

Example: Satellit in orbit
Zentripetalkraft = Zentrifugalkraft
Masses "feel" no weight!

Example:

- Disk: ω=constant, B pushes shot in direction A
- with v'



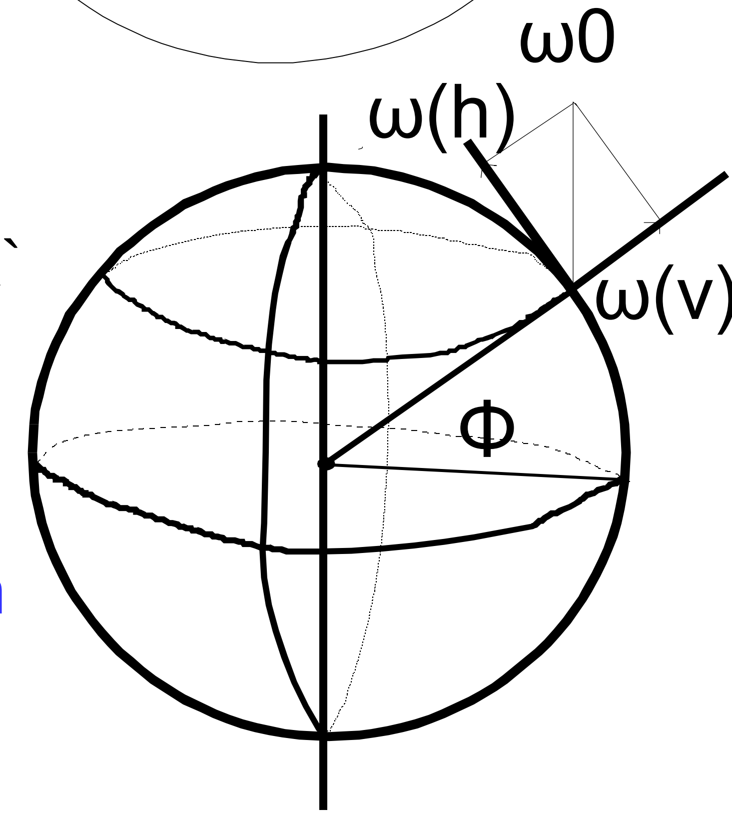
Inertial system: Straight towards C'

After Δt there is a hit at the border!

with Φ =Δt*ω

B moves to B' and expects the hit in A\`

but shot hits at C'-> force ⊥ v'



The earth as a rotating system

1.Movement on surface: ω_v $\vec{F}_c \perp \vec{v}$ *in* plane

Pole: $\boxed{F_c = 2mv\omega_0}$ Generally: $F_C = 2mv\omega_0 \sin\Phi;$

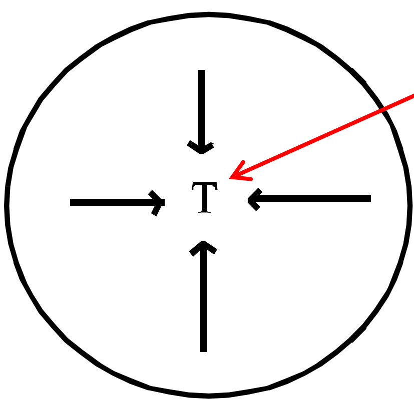
Equator: $F_c = 0$ Φ :

Deflection to the right on north hemishere , deflection to the left auf on south hemisphere!

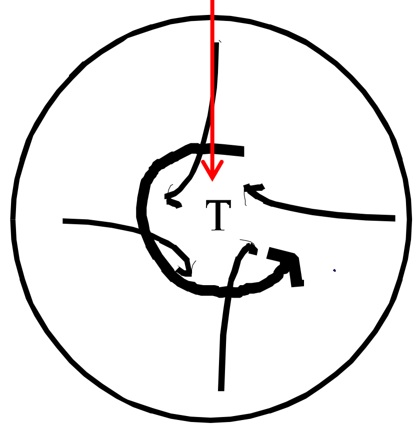
Influence on air going upwards the atmosphere

Low pressure area

1)



$\omega=0$

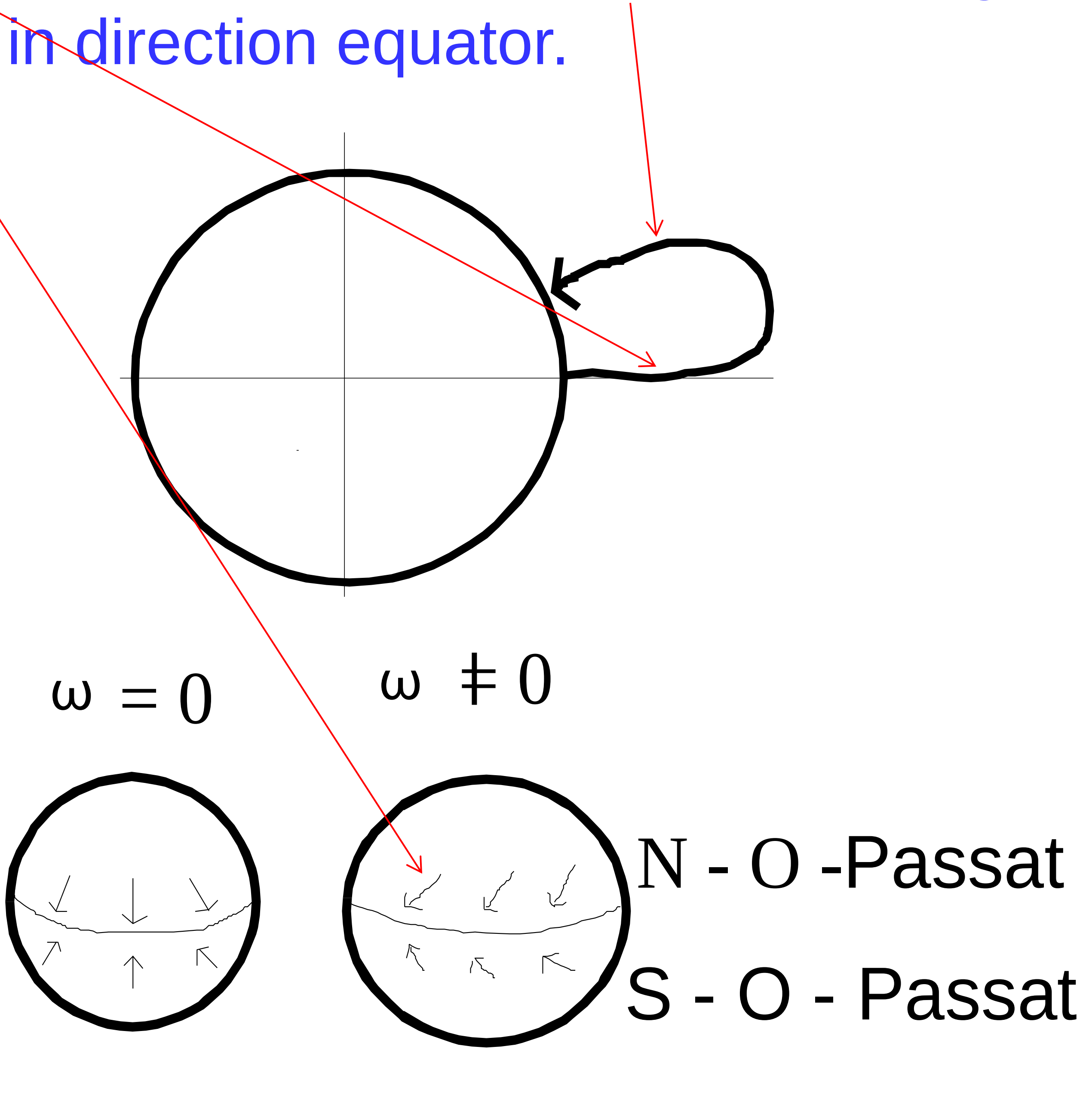


$\omega \neq 0$

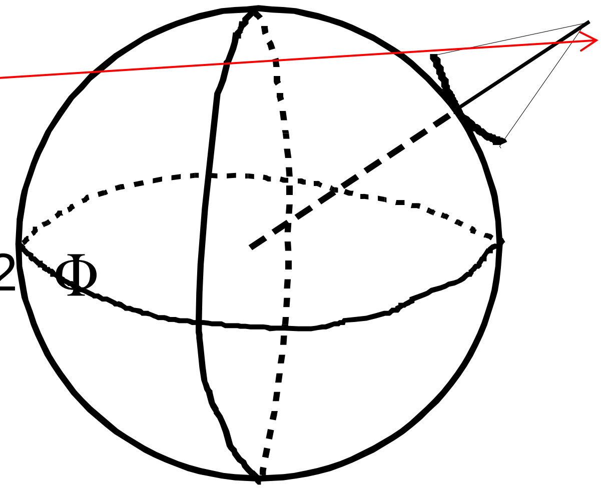
Weather gets determined mostly by Coriolis force

Left hand twist of the airmass

2. Warm air rises and sinks at around 30° again and flows in direction equator.



Proof of rotation of earth with a pendelum (Foucault, Paris 1850)



Inertia: Oscillation plane will be retained!

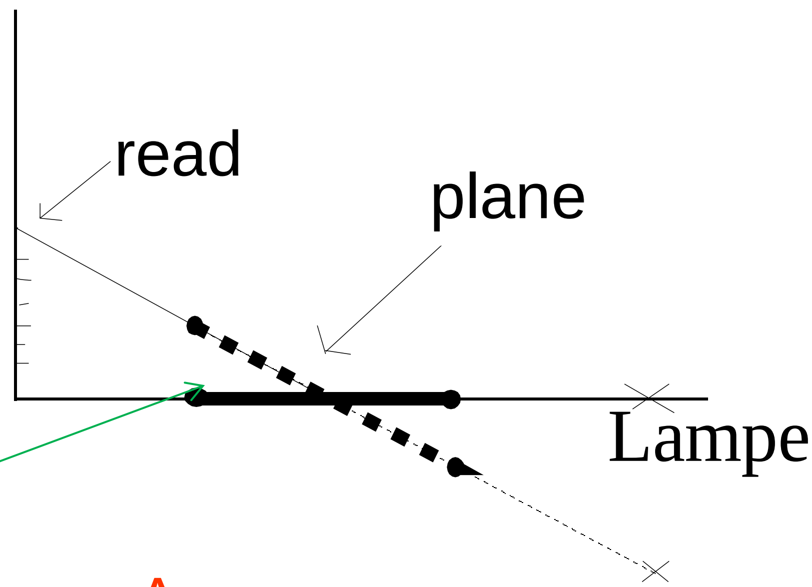
The laboratory rotates around

and rotates in Bonn in T=24/sin =31h; 1min=0.2 degrees

System of inertia: Laboratory

Rotates below oscillation plane. On earth:

Coriolis- Kraft. :Displacement



Pendelum in oscillation plane

$\Delta s = u * \Delta t$ (u Geschwindigkeit)

(b=acceleration due to Coriolis)

Vertical to $\Delta s' = (b/2) * (\Delta t)^{**}2$

Angle of rotation of oscilation plane: $\Delta\alpha = \Delta s' / \Delta s =$

$(u * \omega * \sin\Phi * (\Delta t)^{**}2 / u * \Delta t) = \omega * \sin\Phi * \Delta t$