2.8. Free axes

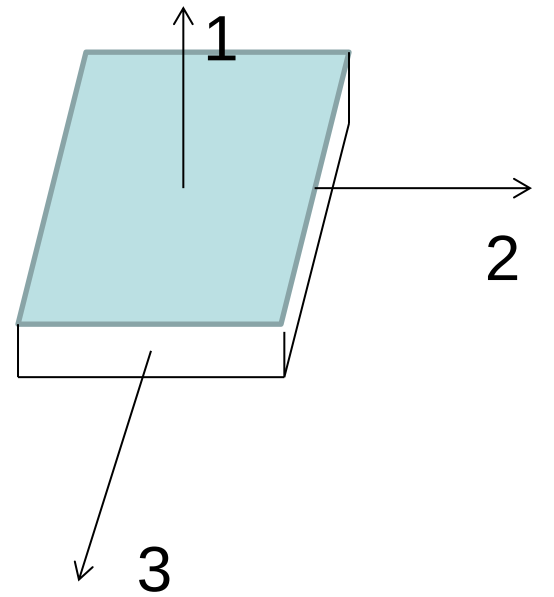
Up til now: Axis of rotation mounted Question: What is the stable axis, as soon a body gets moved to turn ?

Up until now: I = Scalar

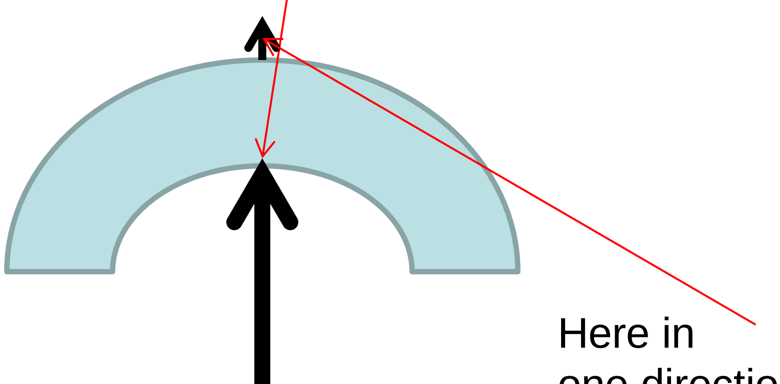
But does not be a scalar

The 3 axes are
Are the main axes of inertia:

$I_1 \quad I_2 \quad I_3$



Example : gyration
(fixed at c.m.),
symmetric spinning top



Here in
one direction

The following axes can
be defined: current axes of rotation

Axis of the figure $\vec{A}$

Axis of rotation $\vec{\omega}$

Axis of angular momentum $\vec{L}$

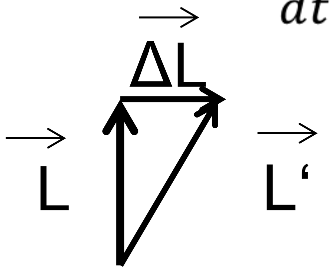
After „to put into rotation “: No $\vec{T}$‘s

$\vec{L} = \overrightarrow{\text{constant}}$ $\qquad \vec{\omega} \parallel \vec{L} \parallel \vec{A}$

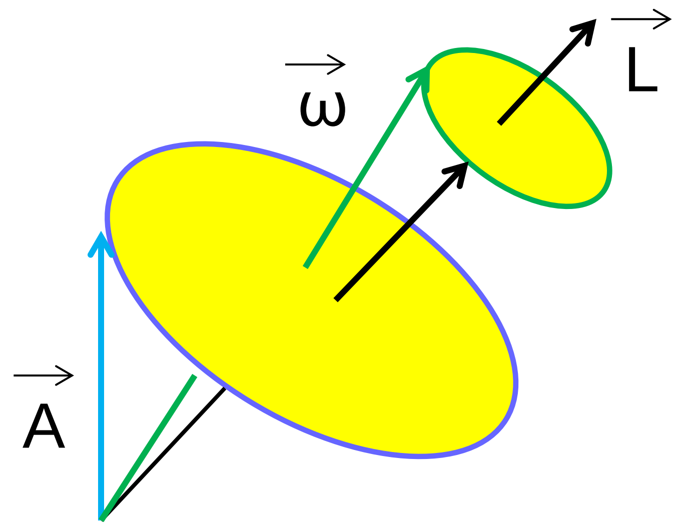
But a small kick against the axis of figure: $\Rightarrow$ Creates T

$\vec{T} = \frac{d\vec{L}}{dt} \Rightarrow d\vec{L} = \vec{T}\,dt$

L-axis gets tilted, all axes have different directions

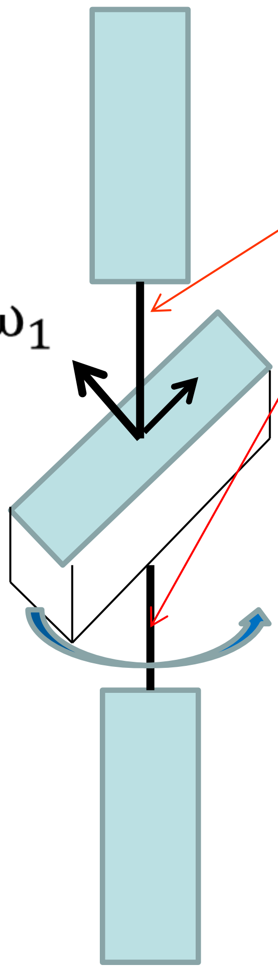


the ω – and the A axis move around the fixed L axis on surface of a cone



Example for the connection of $\vec{L}$ and $\vec{\omega}$:

$L_1 = I_1 * \omega_1$



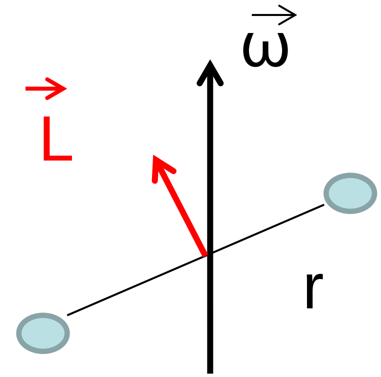
$\vec{\omega}$ – axis fixed mechanically

$L_2 = I_2 * \omega_2$

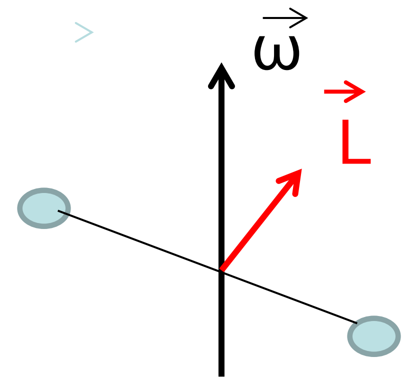
$I_1 \neq I_2 \Rightarrow$

$$\frac{L_1}{L_2} \neq \frac{\omega_1}{\omega_2}$$

Easier to see for two masses



After rotation by 180 deg.

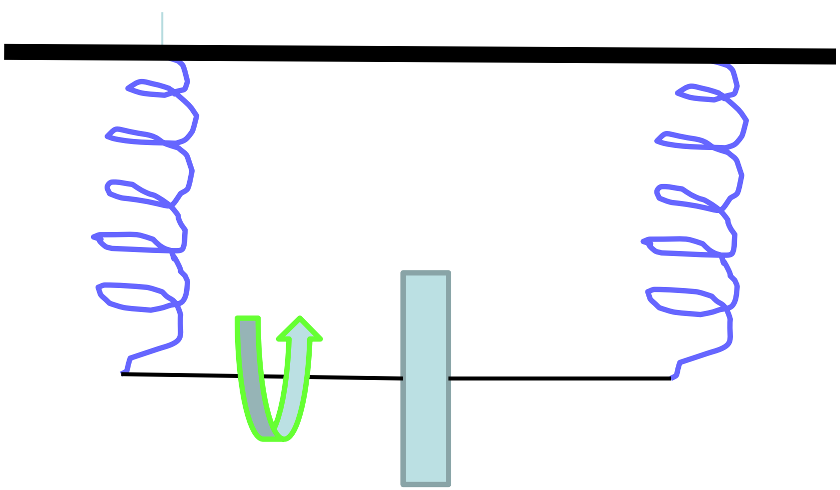


Because $\vec{L} = \vec{r} \times \vec{p}$ $\quad \vec{L} \perp \vec{r}$ $\rightarrow$ L always turns around

$$\rightarrow \quad \frac{d\vec{L}}{dt} = \vec{T} \neq 0 \quad \rightarrow$$

The mount of the set-up have to take up torques ⇒ the run has unbalance!!

To a better demonstration of unbalance an experiment is used with three mobile axes.



To balance out means:

$$\sum_i T_i = 0$$

One has to mount masses in such a way that:

$$\vec{L} = I * \vec{\omega}\ ; \quad \vec{L} \parallel \vec{\omega}$$

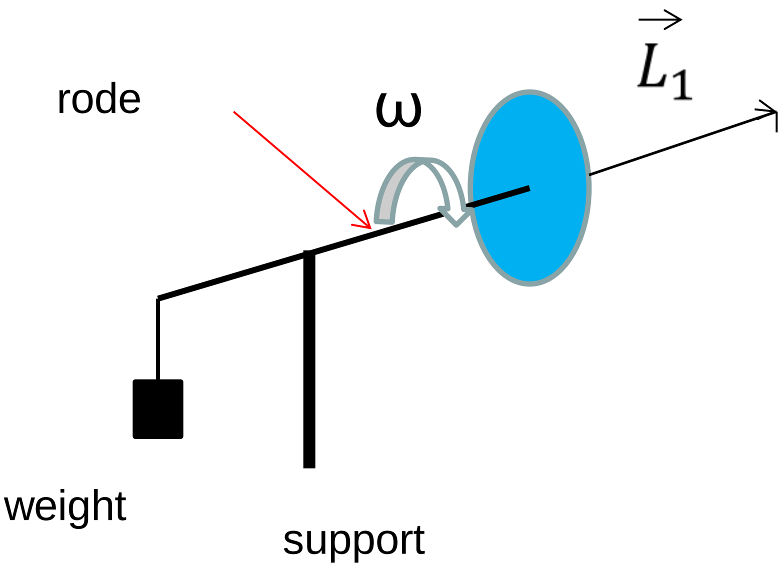
Precession

Spinning tops influenced from outside: Torques change of $\vec{L}$, so far $\vec{T}$ is in action.

a) Direction of $\vec{L}$ remains, then $\vec{T} \parallel \vec{L}$

b) Direction of $\vec{L}$ changes, if $\vec{L} \nparallel \vec{T}$

Example Gyroscope:



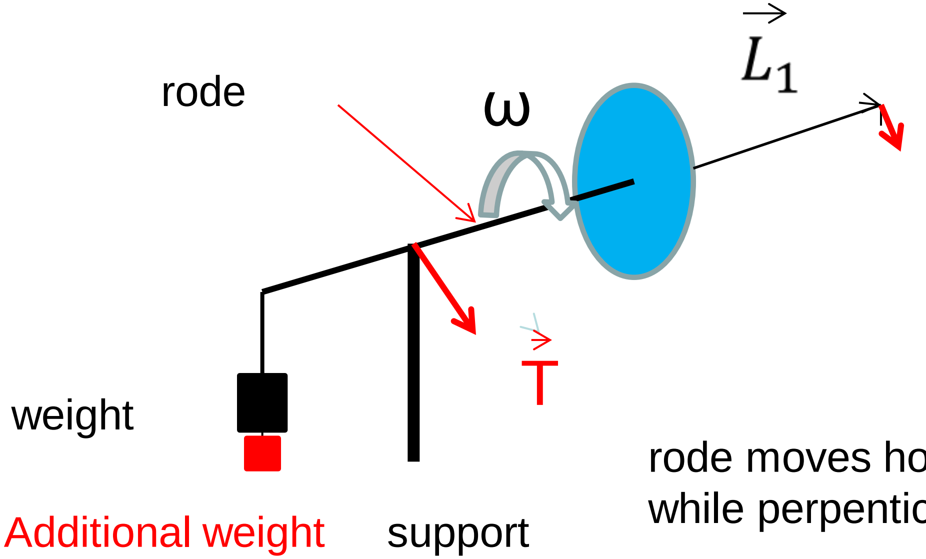
1) $\Sigma\ T_i$ =0

Equilibrium:

$\vec{L}$ = $\overrightarrow{\text{constant}}$

$\Rightarrow$

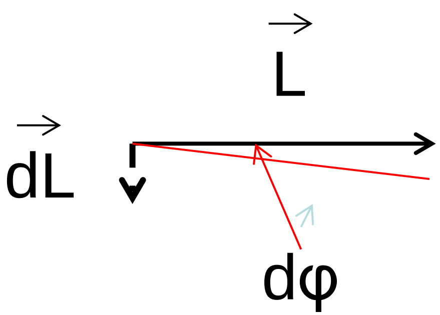
Additional weight causes torque



$$\vec{T} = \frac{\vec{dL}}{dt}$$

rode moves horizontally
while perpenticulary acts the pull

For $\omega_p < \omega$



$$\vec{T} = \frac{\vec{dL}}{dt}$$

$$\vec{dL} = \vec{T} \cdot dt$$

If the additional weight works always: The rode rotates with the frequency of precession

$$dL = d\varphi \cdot L \qquad \frac{dL}{dt} = \omega_p \cdot L$$

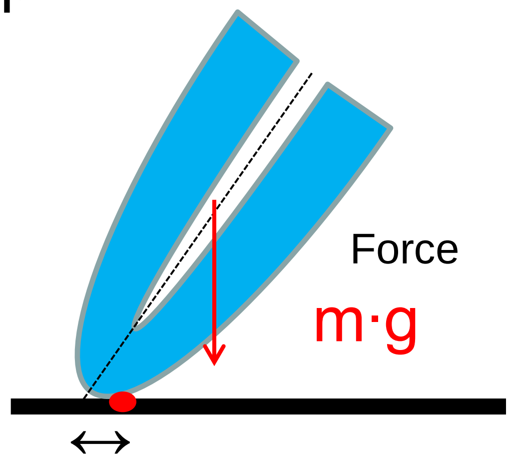
$\vec{T} = \vec{\omega}_p \times \vec{L}$

$$\omega_p = \frac{T}{L}$$

e.g. ω_p: larger, if L smaller

Good to see with the top:
friction causes that L gets smaller > ω_p increases!

Top:



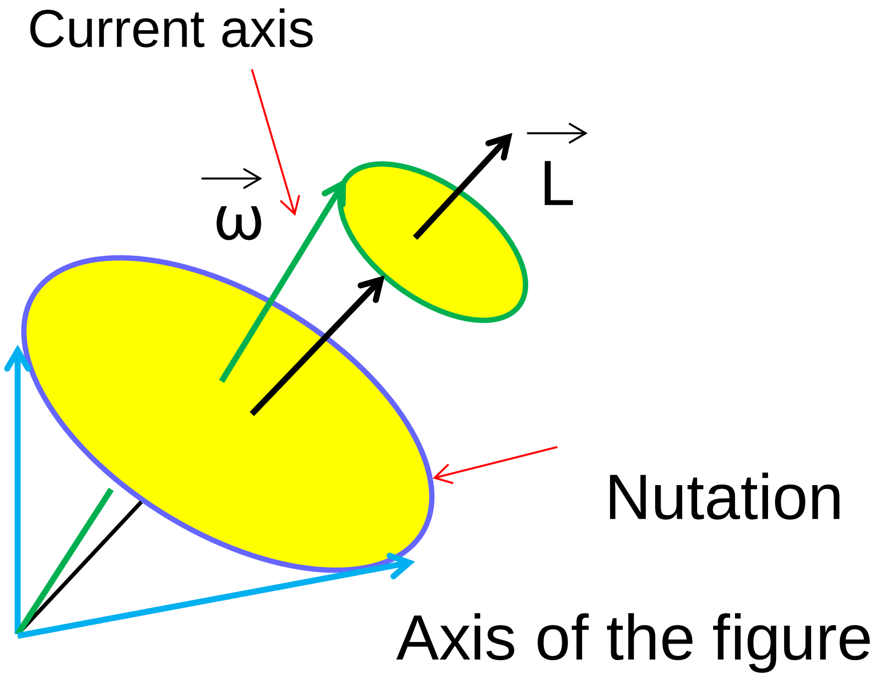
1. Observation: Precession

2. Observation: raising

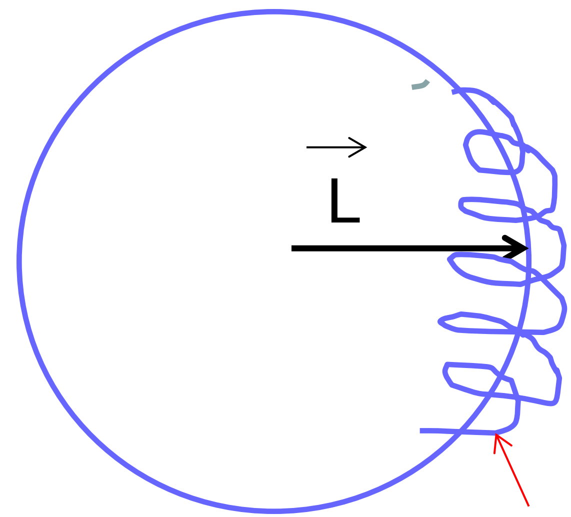
Scrolling from the axis leads torque around c.m.

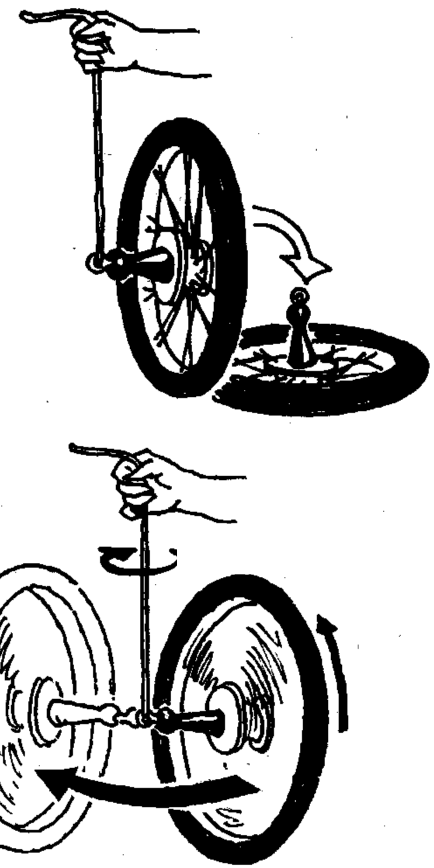
which leads to raising!

Again free axes:



With precession:





Mathematical description of L and ω

A theme for theory