

2.9. Frictional forces

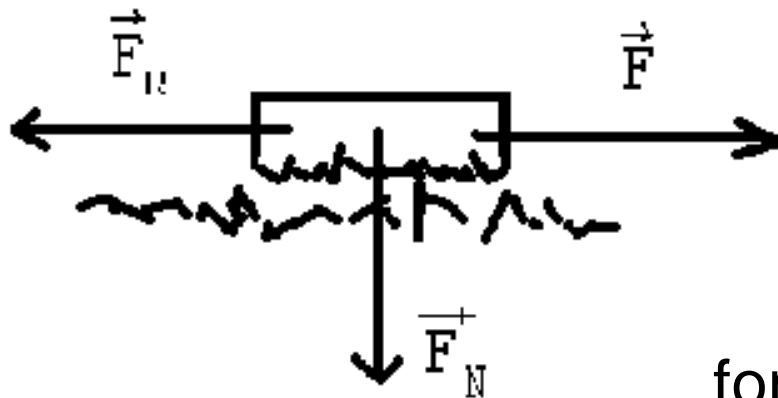
Up until now: Energy conservation

$E_{kin} + E_{pot} = \text{constant}$ **Exemption: Inelastic collision**

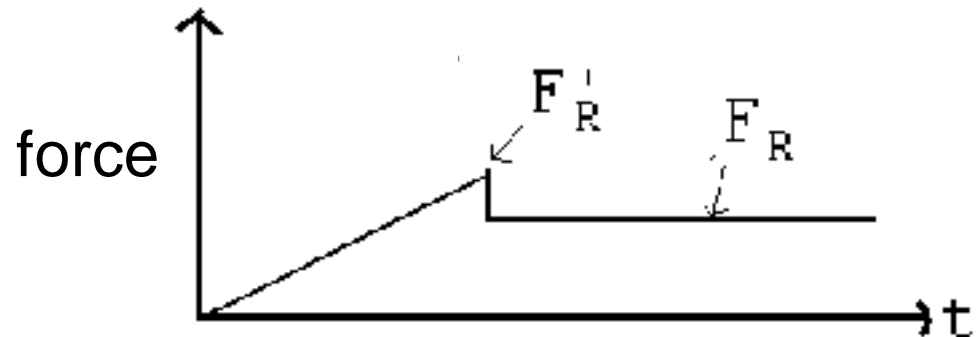
Each real movement faces loss of energy,
e.g. transformation of kinetic energy into thermal energy.

\Rightarrow **frictional forces** \vec{F}_R

Examples: F_R independent of velocity v



At $v=0$: **'sticktion'** F_R'
at $v \neq 0$: **'Slide friction'** \vec{F}_R



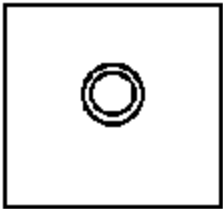
$F_N = F_g$ Normal force (force that presses bodies together)

$$F'_R \sim F_N \quad F_R \sim F_N$$

$$F_R = \mu \cdot F_N$$

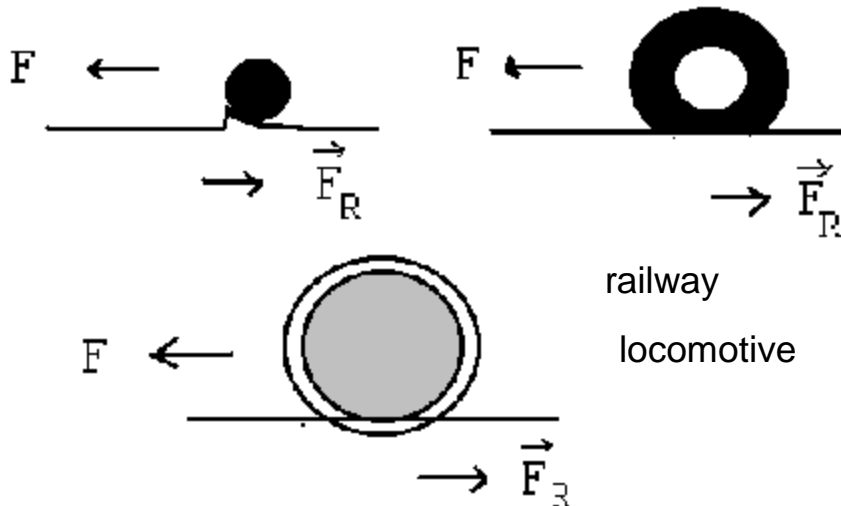
$$F'_R = \mu' \cdot F_N \quad ; \quad \mu' \quad \begin{array}{l} \text{Coefficient of normal force} \\ \text{Coefficient sliding force} \end{array}$$

Very strong surface dependent



μ

Rolling friction at ball bearing
very small
"hard Materials" :
little rolling friction



b) $F_R \sim v$ Stokes or viscous friction

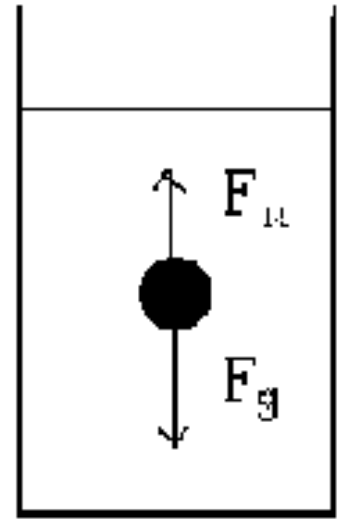
Body in a fluid or gas!

$\vec{F} \sim -\vec{v}$ If v 'strong enough',

$$\text{then } F_R = F_g \Rightarrow m \cdot \ddot{x} = F_g - F_R = 0$$

$$\Rightarrow \ddot{x} = 0$$

\Rightarrow Constant velocity



C) $F_R = k \cdot v^2$ Newton friction

Fast movement through a fluid

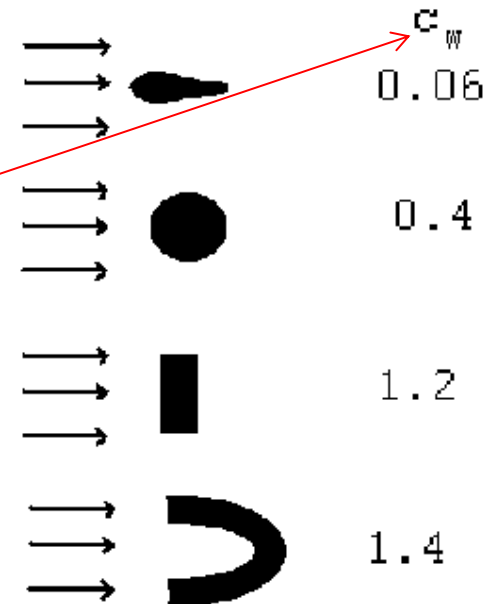
$$k = \frac{1}{2} \cdot c_w \cdot \rho \cdot A$$

c_w : Coefficient of friction:
pure number

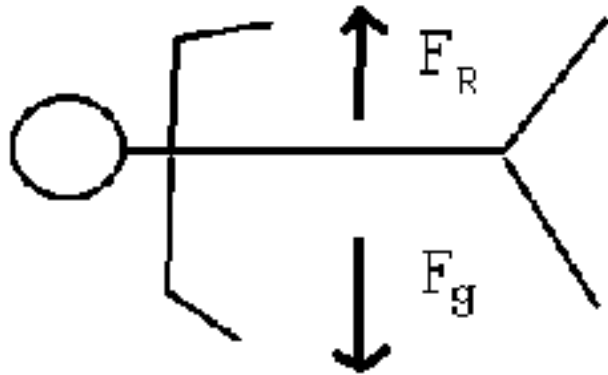
ρ : Density of fluid

A: stream in area

c_w depends on the shape



e.g.: Fall of a human being in a gravitational field with air resistance



Equation of motion:

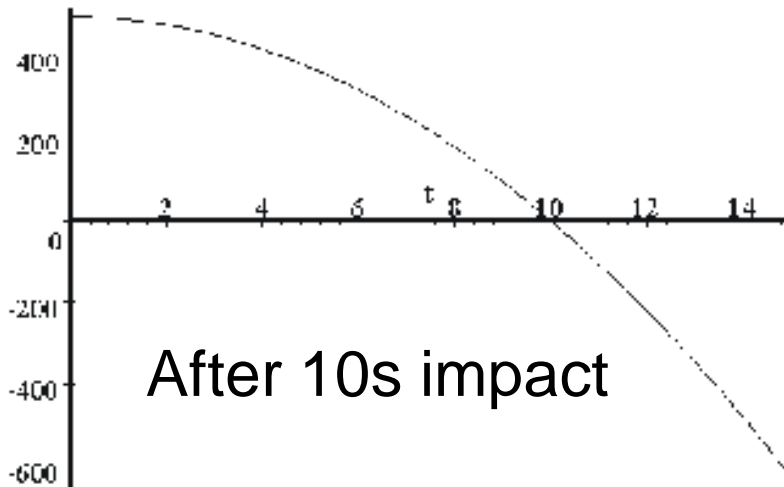
$$m \cdot \ddot{x} = -m \cdot g + k \cdot \dot{x}^2$$

$$\text{If } |\vec{F}_g| = |\vec{F}_R| \Rightarrow \ddot{x} = 0$$

The body sinks with constant velocity v_L :

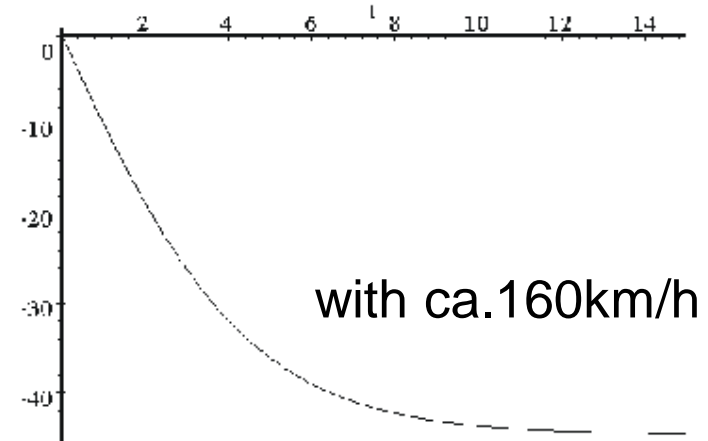
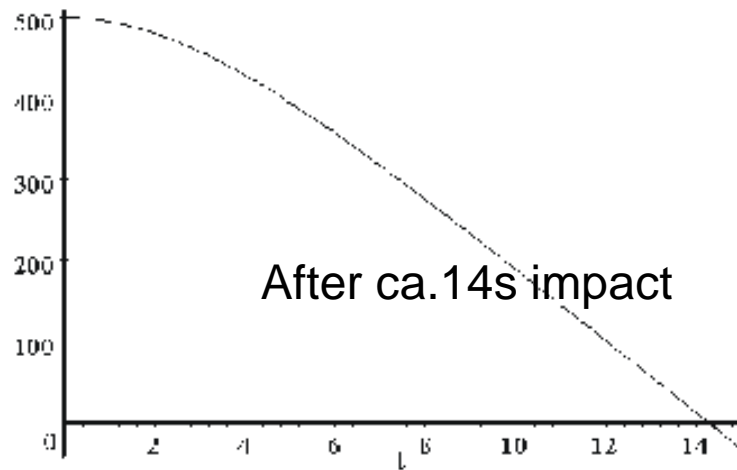
Free fall from
500m(without air):

$$z(t) = -5t^2 + 500$$



Free fall 500m with air: $z(t) = -200.0 \ln(\cosh. 223\ 61t) + 500$

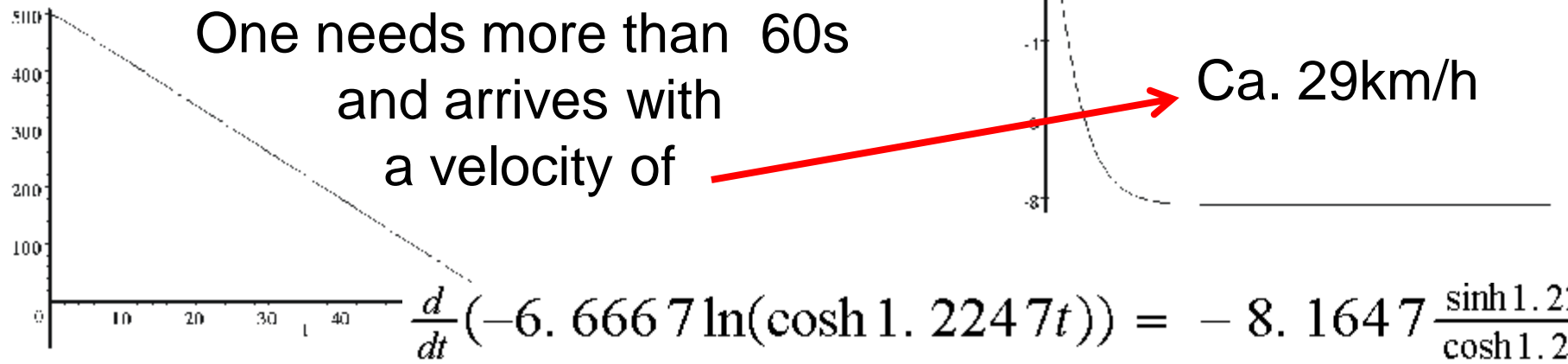
$$\frac{d}{dt}(-200.0 \ln(\cosh. 223\ 61t) + 500) = -44.722 \frac{\sinh. 223\ 61t}{\cosh. 223\ 61t}$$



With parachute $20\ m^2$:

$$\frac{d^2z}{dt^2} - 0.5 \cdot 0.3 \cdot \left(\frac{dz}{dt}\right)^2 + 10 = 0$$

One needs more than 60s and arrives with a velocity of



$$\frac{d}{dt}(-6.6667 \ln(\cosh 1.2247t)) = -8.1647 \frac{\sinh 1.2247t}{\cosh 1.2247t}$$

Remark concerning cars

Tires take part with ca. 20% of friction.

At ca. 50km/h: Rolling friction=air resistance
at 100km/h : Air resistance reaches a value
of around 3-4 times rolling friction