1. Motion of point like bodies

Extension of bodies will be neglected
and mass is concentrated in a point: mass point

1.1 Kinematics of mass points

Movement: Change of position in space as a function of time

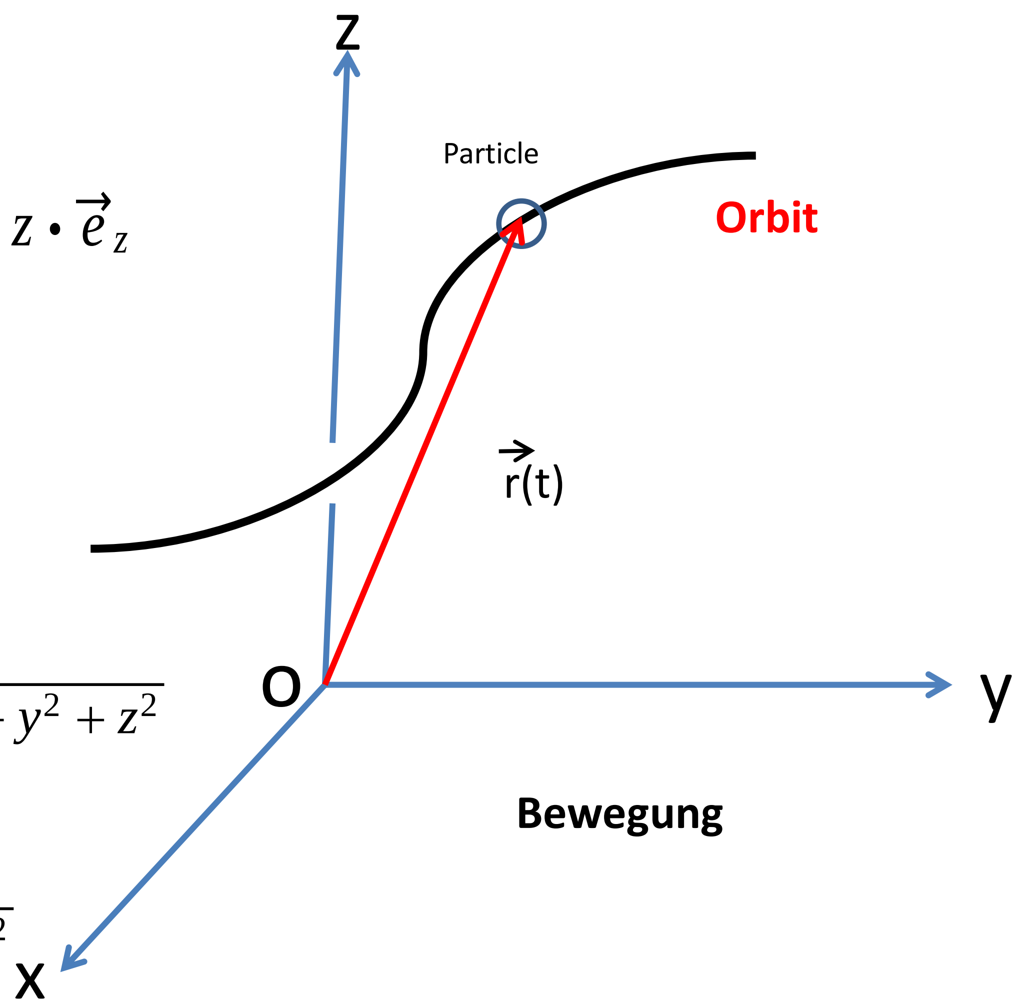
To mark the position of a point in space P(x,y,z)

Coordinates y, x, z

Coordinate system with axes x,y,z

Position vector:

$$\vec{r} = x \bullet \vec{e}_x + y \bullet \vec{e}_y + z \bullet \vec{e}_z$$



$$r = | \; \vec{r} \; | \quad = \sqrt{x^2 + y^2 + z^2}$$

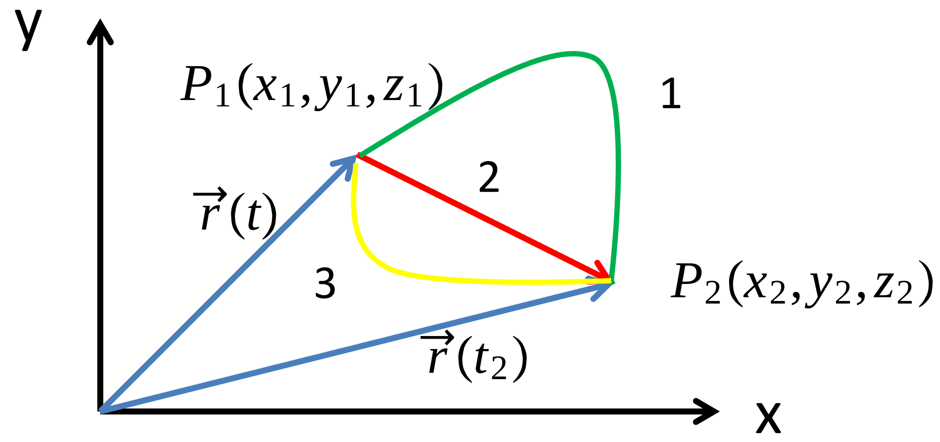
$\vec{r}$ distance from 0

$$r = \left| \vec{r} \right| = \sqrt{x^2 + y^2 + z^2}$$

$\vec{r} = \vec{r}(t); \left| \vec{r} \right|$ function of t

$\vec{r}$: Vector

t : Scalar



1,2,3 Possible **Orbits**

Movement in a straight line: Performed distance

$$\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$$

Δ : means a change of Δ following quantity

Dimension length: Meter (m)

Passed time: $\Delta t = t_2 - t_1$

Dimension time: Secondes (s)

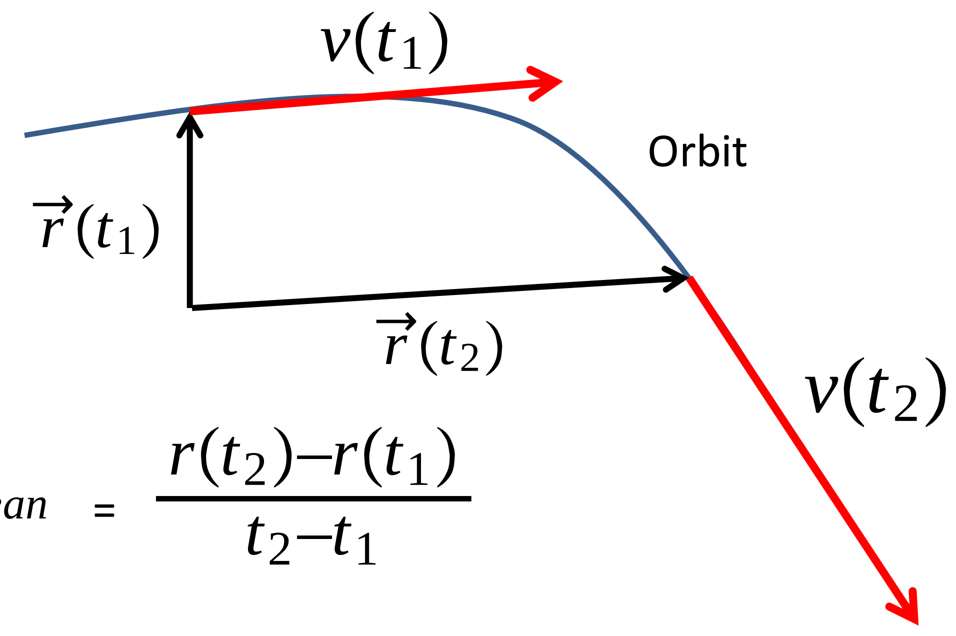
Velocity v:

v=performed distance/passed time

Dimension: meter/second (m/s)

$$\vec{v} = \frac{\overrightarrow{\Delta r}}{\Delta t}$$ Vector into direction $\Delta \vec{r}$

mean **velocity** v_{mean} in $[t_1, t_2]$



$$v_{mean} = \frac{r(t_2) - r(t_1)}{t_2 - t_1}$$

Momentary velocity

$$\vec{v}_1 = \lim_{t_1 \to t_2} \frac{\vec{r}(t_2) - \vec{r}(t_1)}{t_2 - t_1} = \lim_{\Delta t = 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \dot{\vec{r}} = \vec{v}(t)$$

Change of velocity leads to **acceleration**

$$\frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} = \vec{a}(t_1, t_2) \quad \text{with dimension:} \quad \frac{m}{s^2}$$

with $\vec{a}(t_1, t_2)$ as mean acceleration

Momentary acceleration

$$\lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2} = \ddot{\vec{r}}$$

Special cases

a) Change of the amount of $\vec{v}$

Rectilinear move

$$\vec{e}_a = \pm \vec{e}_v \qquad "+" \qquad \text{„acceleration“}$$

$$"-" \qquad \text{„braking“}$$

$$\vec{v}(t) \quad \begin{array}{l} \rightarrow \quad "+" \quad \longrightarrow \qquad \longrightarrow \vec{e}_a \\ \longrightarrow \quad "-" \quad \rightarrow \qquad \longleftarrow \vec{e}_a \end{array}$$

b) Change of direction only $\vec{v}$

$$|\ \vec{v}\ | \quad \text{constant}$$

Up until now $\vec{r}(t) \rightarrow \vec{v}(t) \rightarrow \vec{a}$

Mathematically: Differentiate

The resolution of a problem needs to go very often the other way around

$$\vec{a}(t) \rightarrow \vec{v}(t) \rightarrow \vec{r}(t)$$

Mathematically: Integrate

Example: One dimensional move, e.g.: x-direction

$$a(t) = \frac{dv}{dt} \rightarrow \vec{v}(t) = \int a(t')dt'$$

with $v_0 = v(0)$

$$v(t) = v_0 + \int_0^t a(t')dt'$$

Increase of velocity from t'=0 up to t'=t

Correspondingly: $v(t) \rightarrow x(t)$

$$x(t) = x_0 + \int_0^t v(t')dt'$$

x_0 As starting point at t=0

$\Downarrow$

Increase of distance

$$x(t) = x_0 + \int_0^t \left[v_0 + \int_0^{t'} a(t'')dt'' \right] dt'$$

Example : a(t)=constant = a_0

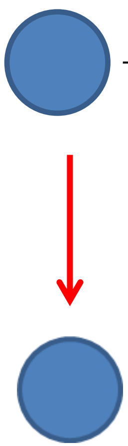
$$v(t) = v_0 + \int_0^t a_0 dt = v_0 + a_0 \cdot t$$

Increases linearly with t

$$x(t) = x_0 + \int_0^t [v_0 + a_0 \cdot t']dt' = x_0 + v_0 \cdot t + \frac{1}{2} a_0 \cdot t^2$$

increases quadratically with t

Example: Free fall



X=0, t=0, V_0=0

+X

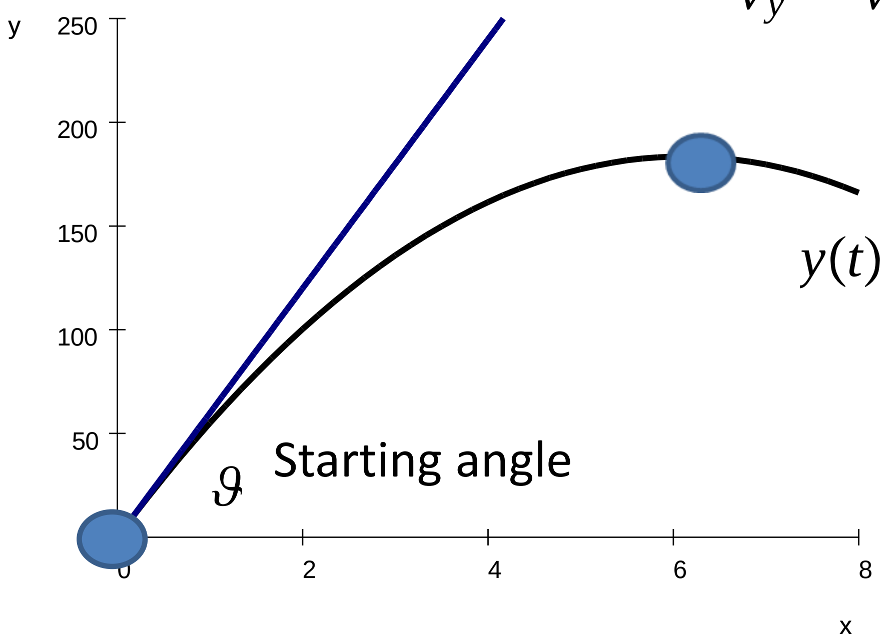
$$x(t) = \frac{1}{2} g \cdot t^2$$ With a_0 =g: g=constant

g: Constant of gravitation

Value surface of the earth

Physics, like e.g. mass dependence of free fall gets discussed later on

Example: für two dimensional orbits



Velocities

$$v_y = v_0 \cdot \sin\vartheta, \quad v_x = \cos\vartheta$$

into y-direction acts g

$$y(t) = y_0 + v_y \cdot t - \frac{1}{2} g \cdot t^2$$

Into x-direction:

$$x(t) = x_0 + v_x \cdot t$$

Start at t=0,x=0,y=0

Equations of movement: $x(t) = v_0 \cdot \cos\vartheta \cdot t, \; y(t) = v_0 \cdot \sin\vartheta \cdot t - \frac{1}{2} g \cdot t^2$

Equation with t as parameter

Elimination of t: Leads to **equation of orbit** in x-y plane

$$t = \frac{x}{v_0 \bullet \cos\vartheta} \longrightarrow y = x \bullet \tan\vartheta - \frac{g \bullet x^2}{2 \bullet v_0} \cos^2\vartheta$$

$$\vartheta = 0 \rightarrow y = -\frac{g \bullet x^2}{2 \bullet v_0^2}$$

Example: Beam of water

Determination of velocity of the beam
at x=y=0
by a measurement of the orbit

Here: $v_0 = 4.43 m/s$