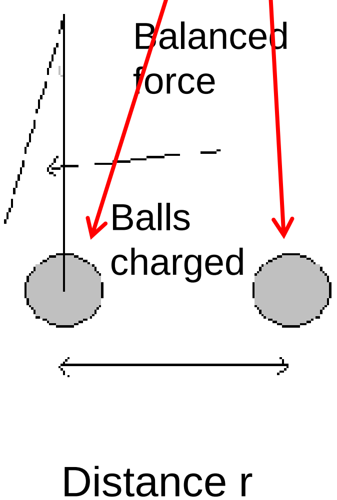
6. Electrostatics

From experiments: rubbing of bars etc.:
Forces: attracting/repulsing
In addition, we observe : sparks, glow, etc.

6.1. Charges and Coulomb-Force

More quantitatively: Experiment



Result: Force

$$\backsim Q_1 \sim Q_2 \backsim \frac{1}{r^2} \ : \ [Q] = Coulomb = 1C, [r] = m$$

$$\Rightarrow K \backsim \frac{Q_1 \cdot Q_2}{r^2}$$

or

$$K = \frac{1}{4\pi \cdot \varepsilon_0} \frac{Q_1 \cdot Q_2}{r^2}$$

Dielectric constant of the Vacuum

$$\varepsilon_0 = 8.85 \cdot 10^{-12} \frac{C^2}{N \cdot m^2}$$

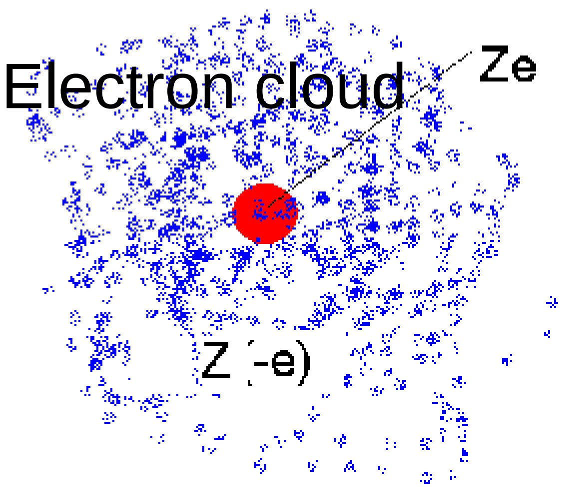
Elementary charge, e.g.: charge of electron: $e = 1.602 \cdot 10^{-19} C$

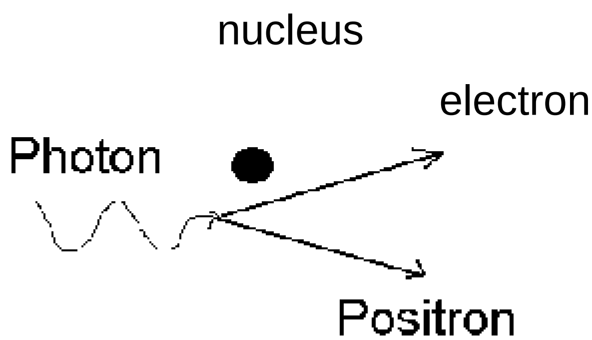
For a classification: more about electric charges:

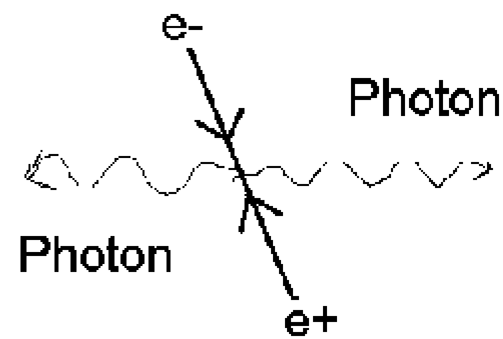
1. Sign: + and -

2. In a closed system: **Conservation of charge**

Atom **Pair production** **Pair annihilation**







3. Multiple of elementary charge → ',quantized"

4. Charge goes always along with mass

5. Electric force beween two point charges works along the direction their connecting line

6. Charges of same sign : repulsive
Opposite charge: attractive

7. Comparison: Electric force compared to force of gravity

Example: 2 Electrons : $q_1 = q_2 = 1.6 \cdot 10^{-19} C$

$m_1 = m_2 = m_0 = 9.1 \cdot 10^{-31} kg$

specific charge

$$\frac{F_C}{F_G} = \frac{\frac{1}{4\pi \cdot \varepsilon_0} \frac{q_1 \cdot q_2}{r^2}}{G \frac{m_1 \cdot m_2}{r^2}} = \frac{1}{4\pi\varepsilon_0 \cdot G} \left(\frac{e}{m_0}\right)^2 : \left(\frac{e}{m_0}\right)$$

$$\frac{F_C}{F_G} = \frac{1.6^2 \cdot 10^{-38}}{4\pi 8.85 \cdot 10^{-12} \cdot 6.67 \cdot 10^{-11} \cdot 9.1^2 \cdot 10^{-62}} = 4.1675 \times 10^{42}$$

Dimensions! o.k.: $\frac{N \cdot m^2 \cdot kg^2 \cdot C^2}{C^2 \cdot N \cdot m^2 \cdot kg^2}$

8. Forces having their origin from charges are additiv

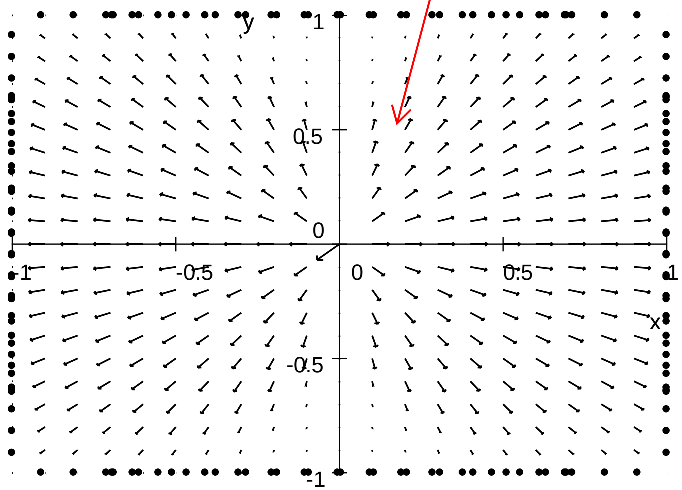
Principle of superposition

6.2.Electric field

Coulombforce:
between q and Q $\overrightarrow{F_C} = \frac{1}{4\pi \cdot \varepsilon_0} \frac{Q}{r^2} \cdot \vec{e}_r \cdot q \; : \; \frac{1}{4\pi \cdot \varepsilon_0} \frac{Q}{r^2} \cdot \vec{e}_r = \vec{E}(r) = \frac{\vec{F}_C}{q}$

$\vec{E}(r)$: Electric field strength $[E] = \frac{N}{C}$

Field of vectors:



In plot we see
"Fieldlines" starting out of Q
Integration covering a surface
holding Q inside:

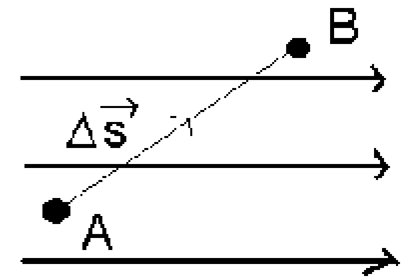
Surface integral yields:

Flux

$$\int \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} = \Phi,$$

Relation due to Gauß

6.3 Electric Potential



$\vec{F} = q_0 \cdot \vec{E}$ (Force on a free charge) produces displacement

$$q_0 : A \rightarrow B$$

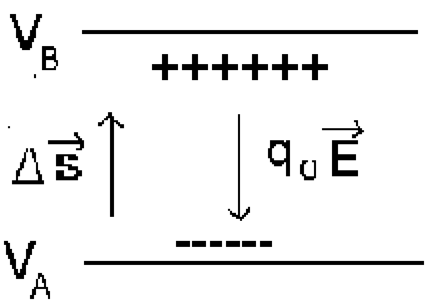
Work will be done:

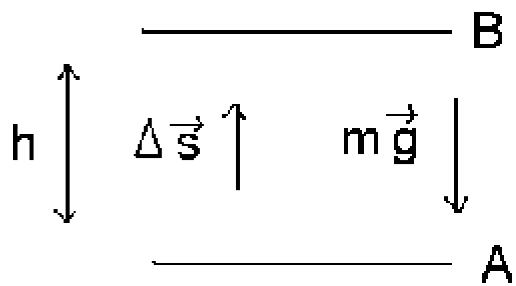
$$W = \vec{F} \cdot \Delta\vec{s} = q_0 \cdot \vec{E} \cdot \Delta\vec{s}$$

Definition: Electric potential difference

$$V_B - V_A = \frac{W}{q_0}$$

(can be be pos. ,neg. or 0)





we compare with Gravitation:

$V_A - V_B = \frac{q_0 \cdot \vec{E} \cdot \Delta\vec{s}}{q_0}$	$V_A - V_B = \frac{m \cdot \vec{g} \cdot \Delta\vec{s}}{m}$
$= -\lvert\vec{E}\rvert \cdot h$	$= -\lvert\vec{g}\rvert \cdot h$
$V_B - V_A = \lvert\vec{E}\rvert \cdot h$	$V_B - V_A = \lvert\vec{g}\rvert \cdot h$

in both cases: Potential in B higher!

$$\frac{W_{AB}}{q_0} = \vec{E} \bullet \Delta\vec{s} = \underbrace{V_A - V_B}$$

Difference of potential between A and B containes properties of the field only

Arbitrary Field: $dW = \vec{F} \bullet d\vec{s} = q_0 \bullet \vec{E} \bullet d\vec{s}$

$$W_{AB} = q_0 \bullet \int_A^B \vec{E} \bullet d\vec{s}$$

$V_B - V_A = -\int_A^B \vec{E} \bullet d\vec{s}$ With the choice $V_A = 0 \Rightarrow V_B$ Is called Potential.

For fields dropping down with ~1/r^2 , one selects V(∞)=0

$\vec{E} \xrightarrow{\text{Integration}}$ V V Differentiation $\vec{E}$; $\boxed{\vec{E} = -\vec{\nabla}(V(r))}$

Potential of a point charge:

Starting point:

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2} \bullet \vec{e}_r$$

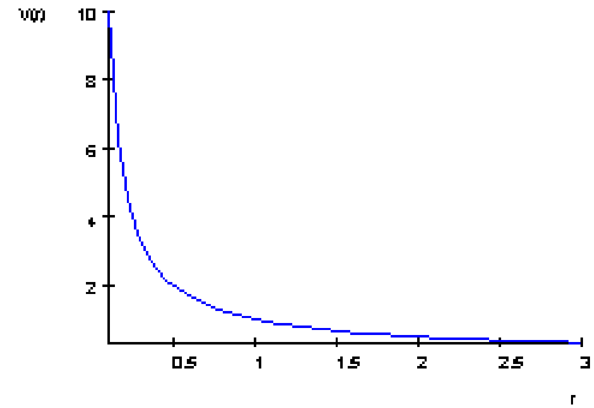
$$V(r_B) - V(r_A) = -\int_{r_A}^{r_B} \vec{E} \bullet d\vec{s}$$

$$-\frac{q}{4\pi\varepsilon_0}\int_{r_A}^{r_B}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

with $V(r_A \to \infty)=0$

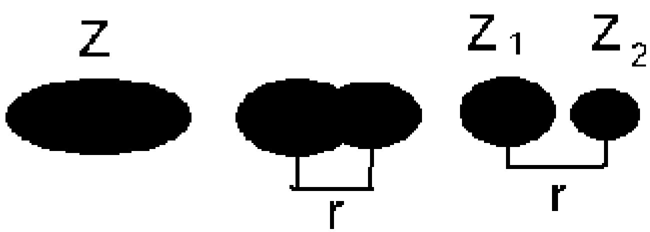
und $r_B = r \Rightarrow$

$$V(r) = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{r}$$ Coulomb-Potential



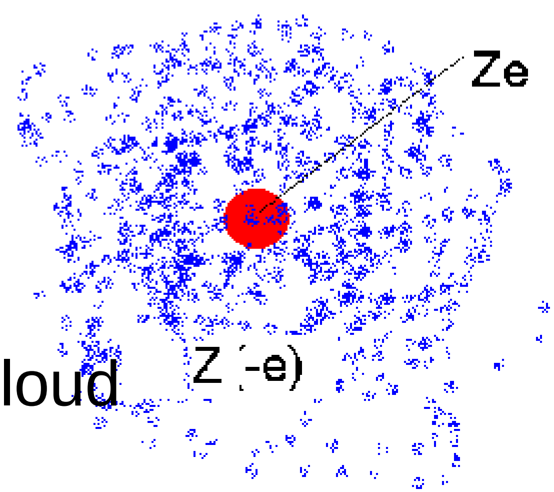
V(r)* e =Potential Energy

e.g: fission of nuclei

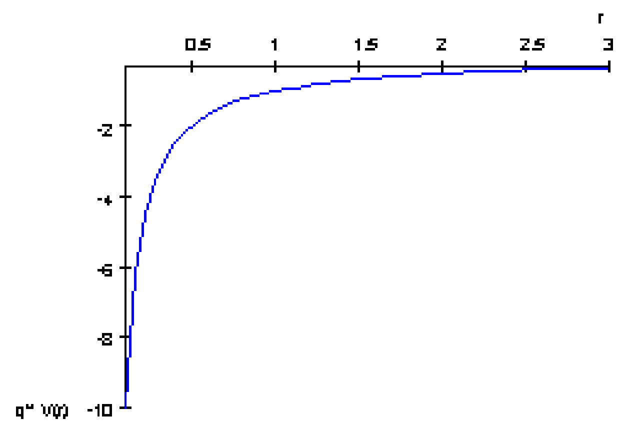


Negative charge (electron) in the field of a pos. charge:

e.g.: Hydrogen atom



Electron cloud



Bohr's radius a_0 of electron distribution in a H-atom :

Estimate

From uncertain relation: P~h/(2π*a)

$$\hbar = 1.05457266 \times 10^{-34}\ \mathrm{J\ s}$$

$$\frac{1}{2} \cdot mv^2 = \frac{p^2}{2m} = \frac{\hbar^2}{2m \cdot a^2}$$

Total Energy of the electron in Coulombfield of the proton

$$E = \frac{\hbar^2}{2m \cdot a^2} - \frac{e^2}{4\pi\varepsilon_0 \cdot a} \Rightarrow$$

Minimal Energy:

$$\frac{dE}{da} = -\frac{\hbar^2}{m \cdot a^3} + \frac{e^2}{4\pi\varepsilon_0 \cdot a^2} = 0 \Rightarrow$$

$$a_0 = \frac{4\pi\varepsilon_0 \cdot \hbar^2}{m \cdot e^2} = \frac{4 \cdot 3.1415 \cdot 8.85 \cdot 10^{-12} \cdot (1.05457266 \times 10^{-34})^2}{9.1 \cdot 10^{-31} \cdot (1.60217733 \times 10^{-19})^2} = 5.2946 \times 10^{-11}\, m$$

Insert into formula for E

$$E_0 = \frac{e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot 2 \cdot a_0} - \frac{e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot a_0} = -\frac{e^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot 2 \cdot a_0}$$

$$= -\frac{(1.60217733 \times 10^{-19})^2}{4 \cdot \pi \cdot \varepsilon_0 \cdot 2 \cdot a_0} = -\frac{(1.60217733 \times 10^{-19})^2}{4 \cdot 3.1415 \cdot 8.85 \cdot 10^{-12} \cdot 2 \cdot 5.2946 \times 10^{-11}} = -2.$$

$$1798 \times 10^{-18} J$$

$$1J = \frac{1}{1.60217733 \times 10^{-19}} = 6.2415 \times 10^{18} eV$$ "Electronvolt"

Energyscale working with atoms!

$$\Rightarrow E_0 = -2.1798 \times 10^{-18} \cdot 6.2415 \times 10^{18} = -13.605 eV$$

Here: Electron bound

For ',separation" from Proton one needs work of 13.6 eV !

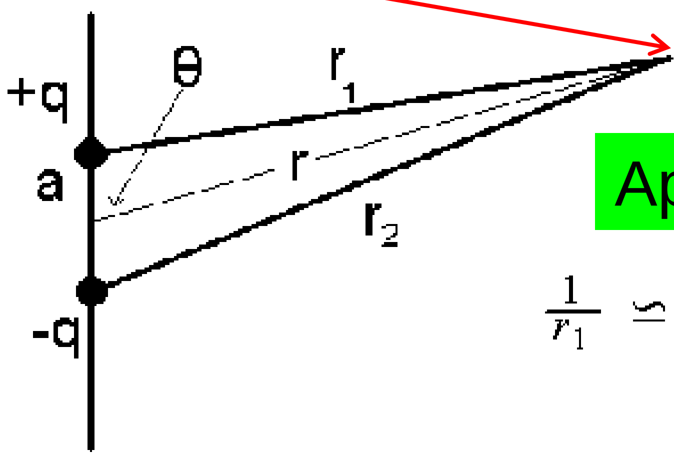
Different materials have different separation energies!

Potential of a Dipole

Superposition principle, Potential Φ

$$\Phi = \sum_i \Phi_i = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

for example:



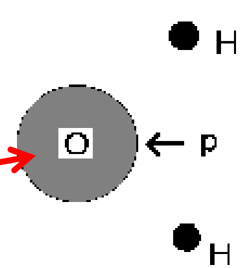
$$\Phi_1 + \Phi_2 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right) = \frac{-1}{4\pi\varepsilon_0}\left(\frac{q\cdot(r_1 - r_2)}{r_1\cdot r_2}\right)$$

Approximation r $\gg 2a$

$$\frac{1}{r_1} \backsimeq \frac{1}{r - a\cdot\cos\theta} = \frac{1}{r(1-\frac{a}{r}\cos\theta)} \backsimeq \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right)$$

In analogy to: $\frac{1}{r_2} \backsimeq \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)$ $\quad \Phi = \frac{1}{4\pi\varepsilon_0}\frac{q\cdot 2a\cdot\cos\theta}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{p\cdot\cos\theta}{r^2}$,

"Dipole": $p = q \cdot 2a$



Electron cloud
most close to oxygen O

e.g.: H_2O: $p = 6.2 \cdot 10^{-30}$ C*m

Dipole:

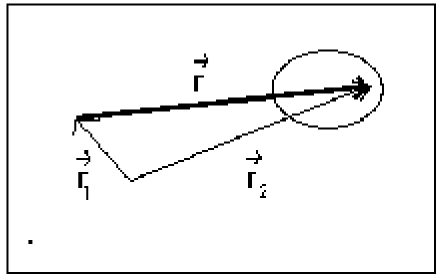
c.m. of
negative and positive
charges do agree!

Orientation! Contineous charge distribution:

a) spatial

$$\rho(r) = \frac{Ladung(dq)}{\text{Volume element}(d\tau)}$$

$$\Phi(\vec{r}_1) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r)d\tau}{r}$$

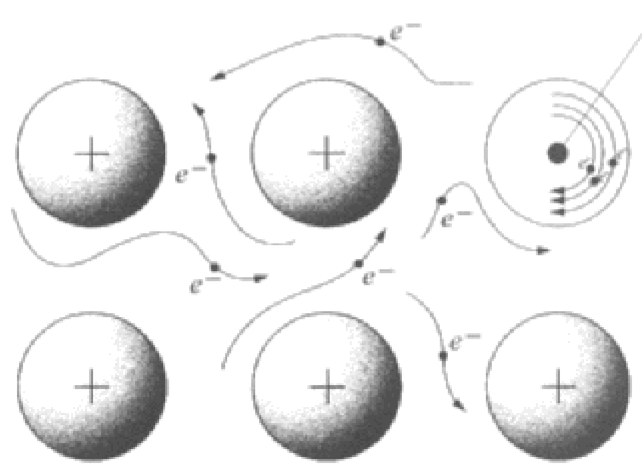


b) flat

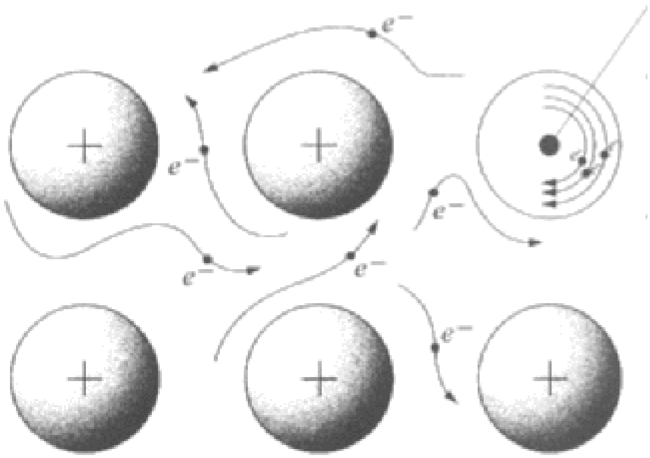
$$\sigma(r) = \frac{\text{charge(dq)}}{\text{surface(dA)}} \qquad \Phi(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(r)dA}{r}$$

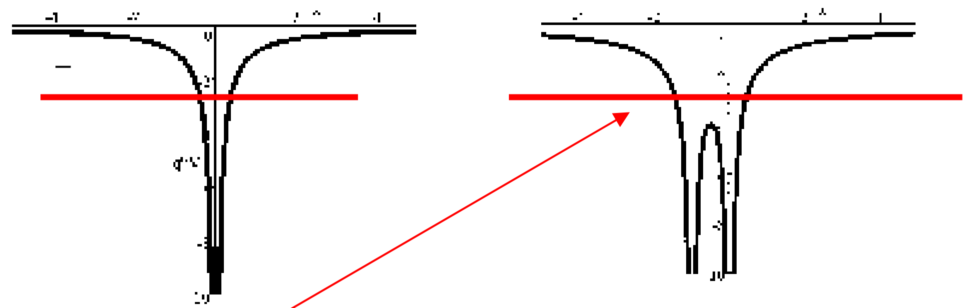
6.4. Electric charges on surfaces of conductors

',Conductor'': Charges are
free movable
To it inspection of
energy of electrons:

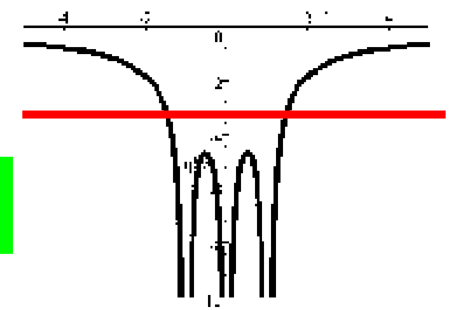


Atomic nuclei: Potential of electrons



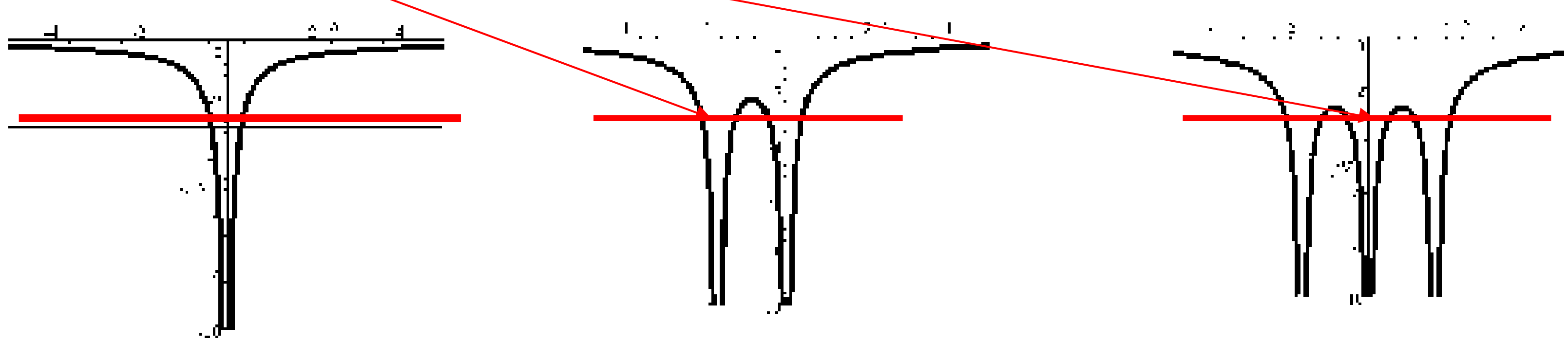


become ',free" movable



Conductor e.g.: Metal

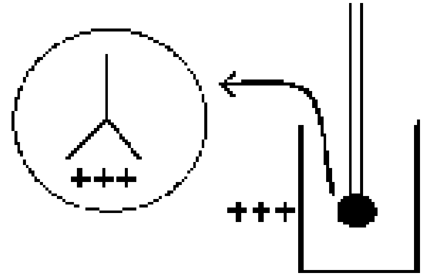
Or stay localized



Isolator

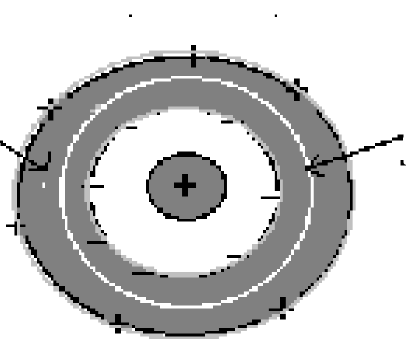
Observations with experiments:

From the inner of a cup
no difference, wether touched
or not!



Charge transfer from a cup!

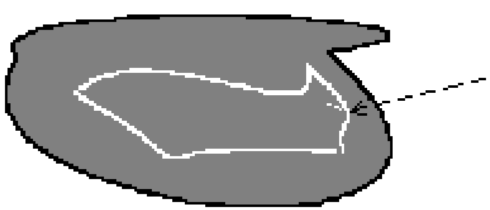
conductor



Conductor $|\vec{E}| = 0$

$$\Rightarrow \int \vec{E} \cdot d\vec{A} = 0! \rightarrow$$

Enclosed charge = 0! Conductor electrostatic equilibrium



Gauß-
surface

$$\int_A \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0} = 0$$

$|\vec{E}| = 0$ im Inneren

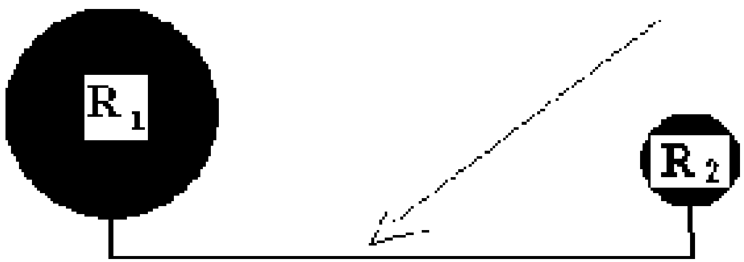
otherwise moving charges $\Rightarrow$ current

Each excess charge has to be on surface!

Thereby $\vec{E}$ stands perpendicular on surface!

Distribution of charge on surface:
Example balls:

Conducting connection $\Longrightarrow$



same potential
via earth

$$\varphi(R) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R};$$

connection: $\varphi(R_1) = \varphi(R_2) \Rightarrow \dfrac{Q_1}{R_1} = \dfrac{Q_2}{R_2}$

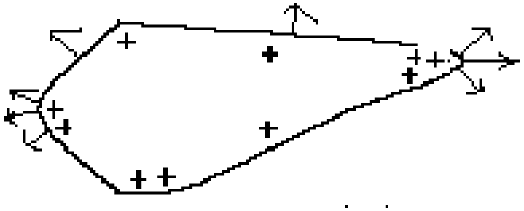
Surface density:

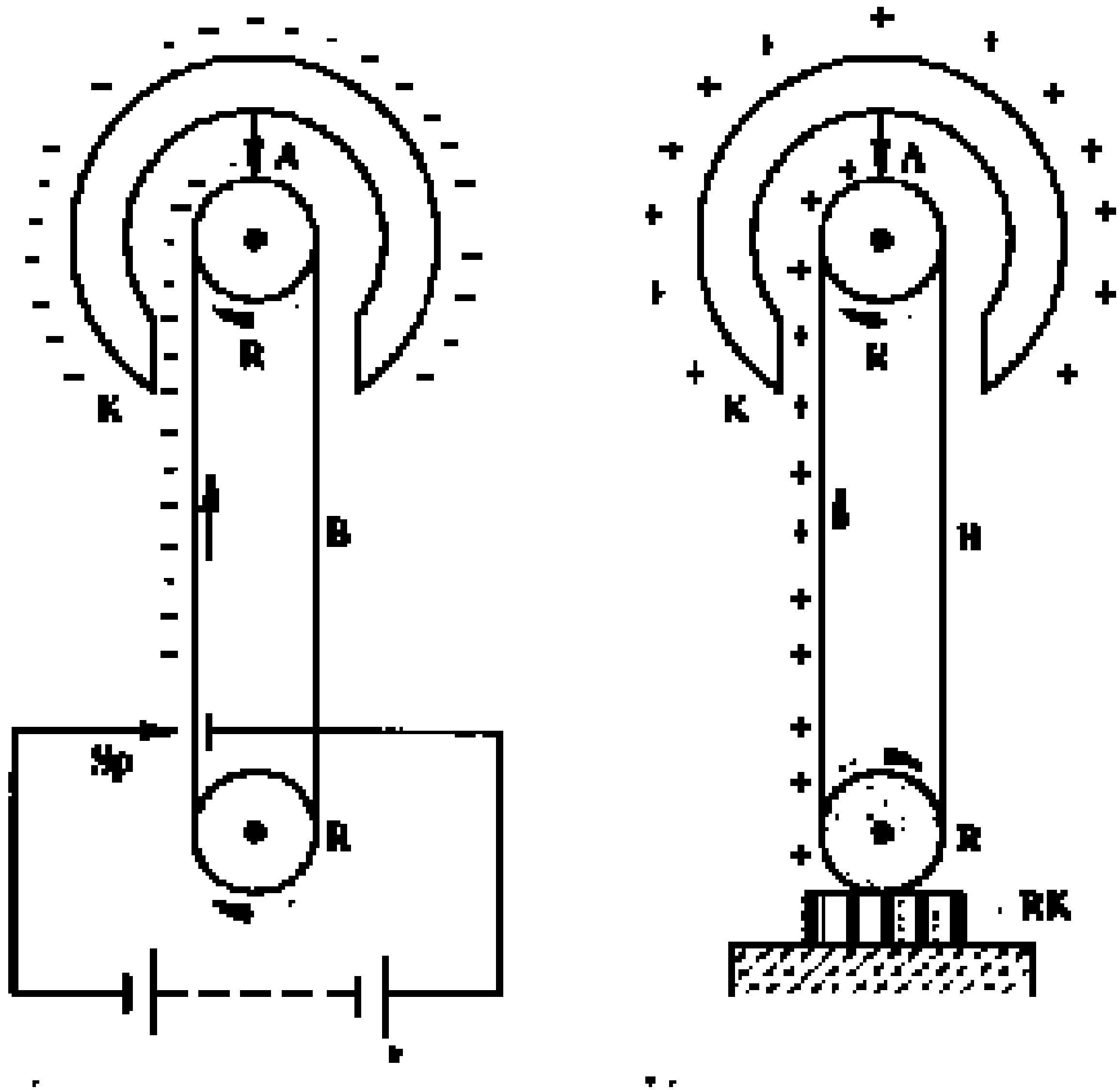
$$\sigma = \frac{Q}{4\pi R^2} \Longrightarrow \frac{Q_1}{4\pi R_1^2}, \frac{Q_2}{4\pi R_2^2}$$

with $\dfrac{Q_1}{R_1} = \dfrac{Q_2}{R_2} \Longrightarrow \dfrac{R_1 \cdot Q_1}{R_1^2} = \dfrac{R_2 \cdot Q_2}{R_2^2}$ or $\dfrac{R_1 \cdot Q_1}{4\pi R_1^2} = \dfrac{R_2 \cdot Q_2}{4\pi R_2^2} \Longrightarrow \dfrac{\sigma_1}{\sigma_2} = \dfrac{R_2}{R_1}$

Charge densities act
like inverse
to radius of curvature!

Example:van de Graaf



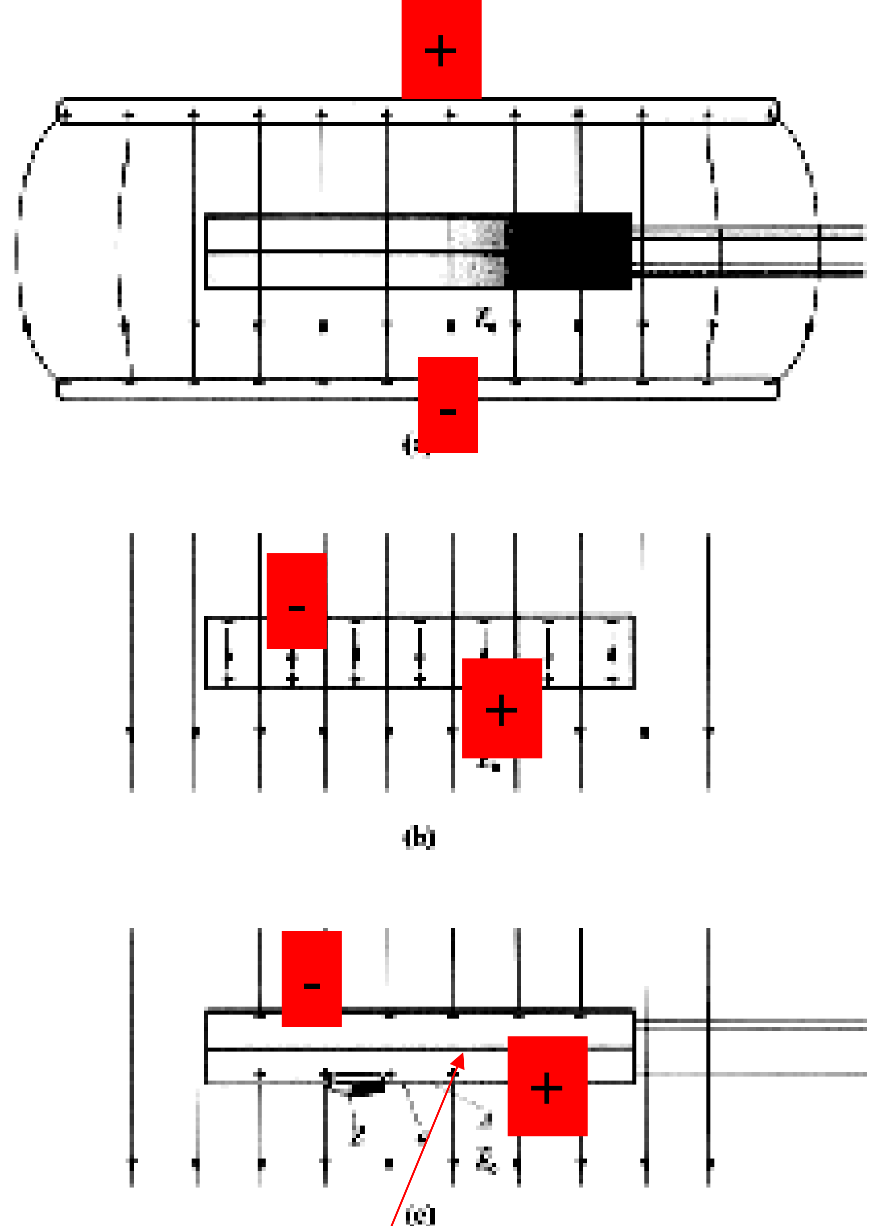


6.5 Induction

Out of following demonstration:

Induced charges can
Be seperated!

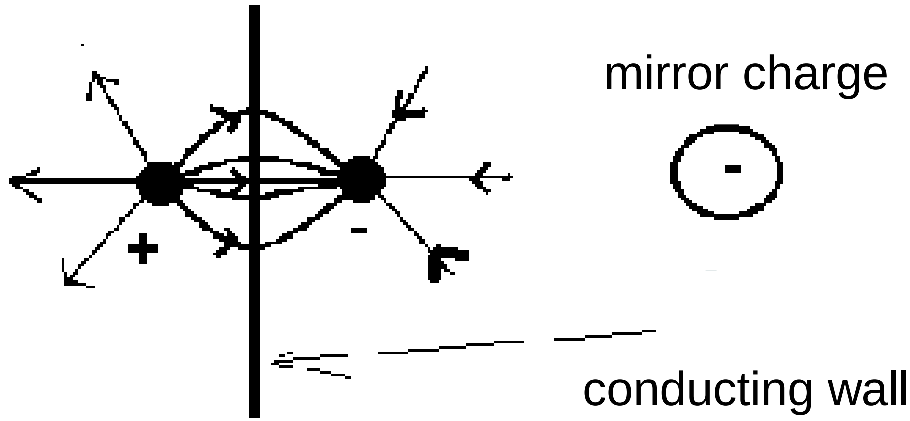
+



trennen

Again induced charges can be separated!

Point charge in front of a wall of metal



Field lines stand perpendicular !
on conduction surface

Force on a positive charge:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2d)^2}$$

D: distance of charge from wall

Image force